Objective: Reduce rational expressions completely.

Concept

A **rational expression** is a ratio of polynomials, $\frac{p(x)}{q(x)}$, where the polynomial in denominator contains at least one variable. **A rational expression is undefined for real values of the variable(s) which create values of 0 in the denominator,** since division by 0 is undefined.

excluded values

Examples of Rational Expressions: $\frac{x+3}{xy-5y}$, $\frac{4}{9x}$, $\frac{x^2-4x+8}{x^3-8}$

Non-Examples: $\frac{6x+3}{2}$ (denominator doesn't contain a variable)

$\frac{1}{2}x^2 - \frac{3}{2}x$ (this is a binomial with rational coefficients)

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Concept

Rational expressions in simplest form cannot contain any common factors between the numerator and denominator.

Steps to Reduce a Rational Expression

1. Factor the numerator and denominator completely.

2. Reduce common factors.

3. Multiply remaining factors so the numerator and denominator are in standard form.

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Ex) Simplify (b) each expression. State the excluded values (a).

$$\frac{x-6}{x^2-5x-6}$$

$(x-6)(x+1)$

Note: The excluded values should be for the original expression.

(a) excluded values

$x-6=0$, $x+1=0$

$x \neq 6$ $\quad x \neq -1$

or

$x \neq -1, 6$

(b) simplify

$$\frac{(x-6)}{(x-6)(x+1)}$$

$$\Rightarrow \frac{1}{x+1}$$

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Ex) Simplify each expression. State the excluded values.

Binomials which are opposites reduce to -1.
Opposite binomials will be of the **form** $ax - b$ **and** $b - ax$.

$$\frac{6-2x}{x^2-9}$$

$(x+3)(x-3)$

ⓐ excluded values

$x+3=0$ $\quad x-3=0$

$x \neq -3$ $\quad x \neq 3$

$x \neq -3, 3$

ⓑ simplify

$$\frac{2(3-x)}{(x+3)(x-3)}$$

$(3-x)$ opposites $(x-3)$, -1

$$\frac{-2}{x+3}$$

or

gcf $= -2$

$$\frac{-2(-3+x)}{(x+3)(x-3)}$$

$$= \frac{-2(x-3)}{(x+3)(x-3)}$$

$$= \frac{-2}{x+3}$$

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Ex) Simplify each expression. State the excluded values.

$$\frac{x^3+8x}{x^4-64}$$

$(x^2+8)(x^2-8)$

ⓐ excluded values

$x^2+8=0$, $x^2-8=0$

$x^2=-8$ $\quad x^2=8$

$\sqrt{x^2}=\pm\sqrt{-8}$ $\quad \sqrt{x^2}=\pm\sqrt{8}$ $\quad \sqrt{4}\cdot\sqrt{2}$

$x=\pm\sqrt{8}i$ imaginary $\quad x\neq\pm 2\sqrt{2}$

$x\neq -2\sqrt{2}, 2\sqrt{2}$

ⓑ simplify

$$\frac{x(x^2+8)}{(x^2+8)(x^2-8)}$$

(1) (1)

$$\frac{x}{x^2-8}$$

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Ex) Simplify each expression. State the excluded values.

$$\frac{x^3+27}{x^2+3x}$$

sum of cubes
$(x)^3+(3)^3$
SOAP

$x(x+3)$

(a) excluded values

$x=0$, $x+3=0$

$x\neq 0$ $\quad x\neq -3$

$x\neq -3, 0$

(b) simplify.

$$\frac{(x+3)(x^2-3x+9)}{x(x+3)}$$

$$\frac{x^2-3x+9}{x}$$

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Ex) Simplify the expression.

$$\frac{x^3 - x^2 - 4x + 4}{4x^2 + 8x}$$

$4x(x+2)$

(a) excluded values

$\frac{4x}{4} = \frac{0}{4}$, $x + 2 = 0$

$x \neq 0$ $\quad x \neq -2$

$x \neq -2, 0$

(b) simplify

$$\frac{x^2(x-1) - 4(x-1)}{4 \cdot x \cdot (x+2)}$$

$$\frac{(x-1) \cdot (x^2-4)}{4 \cdot x \cdot (x+2)}$$

$$\frac{(x-1)(x+2)(x-2)}{4 \cdot x \cdot (x+2)}$$

$$\frac{x^2 - 3x + 2}{4x}$$