## Concept

A <u>rational expression</u> is a ratio of polynomials,  $\frac{p(x)}{q(x)}$ , where the polynomial in denominator contains at least one variable. A <u>rational expression</u> is undefined for real values of the variable(s) which create values of 0 in the denominator, since division by 0 is undefined.

Examples of Rational Expressions: 
$$\frac{x+3}{xy-5y}$$
,  $\frac{4}{9x}$ ,  $\frac{x^2-4x+8}{x^3-8}$ 

Non-Examples: 
$$\frac{6x+3}{2}$$
 (denominator doesn't contain a variable)  $\frac{1}{2}x^2 - \frac{3}{2}x$  (this is a binomial with rational coefficients)



## Concept

Rational expressions in simplest form cannot contain any common factors between the numerator and denominator.

## **Steps to Reduce a Rational Expression**

- 1. Factor the numerator and denominator completely.
- 2. Reduce common factors.
- 3. Multiply remaining factors so the numerator and denominator are in standard form.



Ex) Simplify each expression. State the excluded values.

$$\frac{x-6}{x^2-5x-6}$$

$$(x-\omega)(x+1)$$

Note: The excluded values should be for the original expression.

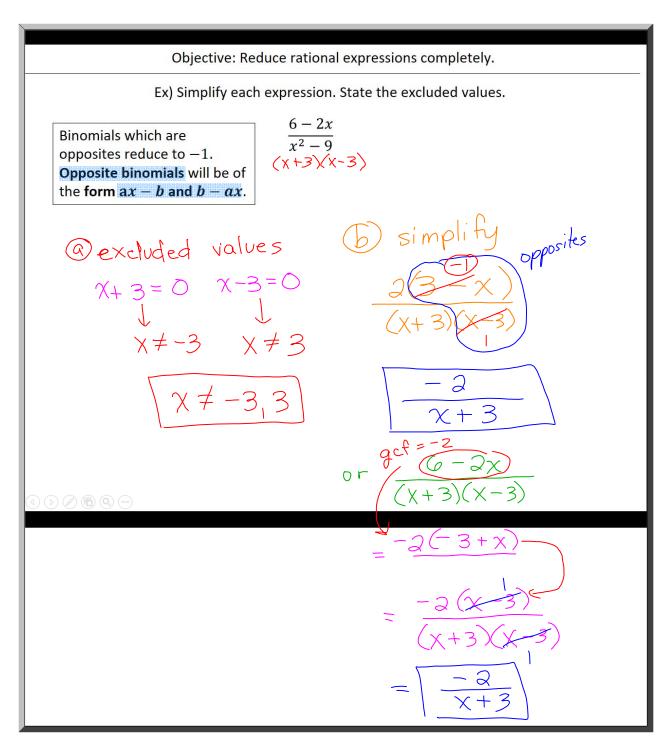
@excluded values

$$\chi - \phi = 0, \quad \chi + 1 = 0$$

$$\sqrt{\chi \neq -1,6}$$

$$\frac{(x-6)}{(x+1)}$$

$$\Rightarrow \boxed{\frac{1}{\chi + 1}}$$



Ex) Simplify each expression. State the excluded values.

$$\frac{x^{3} + 8x}{x^{4} - 64}$$

$$(x^{2} + 8)(x^{2} - 8)$$

@excluded values

$$\chi^{2} + 8 = 0$$
,  $\chi^{2} - 8 = 0$   
 $\chi^{2} = 8$   
 $\chi^{2} = 8$   
 $\chi^{2} = 4$   
 $\chi^{2} = 8$   
 $\chi^{2} = 4$   
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 $\chi^{2} = 4$   
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 $\frac{5}{x} \frac{\sin \beta \sin \beta}{x^2 + 8}$   $\frac{x^2 + 8}{x^2 + 8} (x^2 - 8)$ 



Ex) Simplify each expression. State the excluded values.

$$\frac{x^{3} + 27}{x^{2} + 3x}$$

$$\chi(\chi)^{3} + (3)$$

$$50AP$$

@excluded values

$$\chi = 0, \chi + 3 = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$\chi \neq 0 \qquad \chi \neq -3$$

$$\chi \neq -3,0$$

$$(x+3)(x^2-3x+9)$$
  
 $(x+3)$   
 $(x+3)$   
 $(x+3)$   
 $(x+3)$   
 $(x+3)$   
 $(x+3)$ 



Objective: Reduce rational exp	pressions completely.
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Ex) Simplify the expression.

$$\frac{x^3 - x^2 - 4x + 4}{4x^2 + 8x}$$

$$\frac{4}{2} \left( \frac{1}{2} + \frac{1}{2} \right)$$

$$\frac{4x^{2} + 8x}{4x(x+2)}$$
© excluded values
$$\frac{4y}{4} = 0, \quad x+2=0$$

$$\frac{4x^{2} + 8x}{4x(x+2)}$$
b) simplify
$$\frac{4x^{2} + 8x}{4x(x+2)}$$

$$\frac{4x}{4} = 0, \quad x+2=0$$

$$\frac{4x^{2} + 8x}{4x(x+2)}$$

$$\frac{x^{2} + x^{2} + x^{2}}{4x(x+2)}$$

$$\frac{x^{2} + x^{2}}{4x(x+2)}$$

$$\sqrt{\chi \neq -2,0}$$

$$\frac{\chi^{2}(\chi-1)-4(\chi-1)}{4\cdot\chi\cdot(\chi+2)}$$

$$\frac{(\chi-1)\cdot(\chi^2-4)}{4\cdot\chi\cdot(\chi+2)}$$

$$\frac{\chi^2 - 3\chi + 2}{4\chi}$$