

Objective: Solve rational equations algebraically.

Concept

Rational Equation: an equation that involves at least one rational expression (the variable is in at least one denominator).

Examples

$$\frac{4}{5x} - \frac{2}{7} = \frac{1}{x} \quad \frac{3x}{x+1} = \frac{2}{x-2} + \frac{x-4}{x^2-x-2} \quad \frac{5}{x^2-49} = \frac{x+2}{x-7}$$

Non-Examples

$$\frac{4}{x+5} - \frac{2}{7x} + \frac{1}{x} \quad (\text{no equal sign})$$

$$\frac{3x}{2} = \frac{x+2}{3} + \frac{x-4}{5} \quad (\text{no variable in the denominator})$$



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Rational equations can have **extraneous solutions**: solutions that **are excluded values of the expressions in the equation**. For a rational equation, **extraneous solutions are values that create a 0 in one or more denominator, making that expression undefined**. Extraneous solutions are not included in the final solution set.

For $\frac{4}{5x} - \frac{2}{7} = \frac{1}{x}$, a solution of 0 would be extraneous and would not be included in the final solution set.

For $\frac{3x}{x+1} = \frac{2}{x-2} + \frac{x-4}{x^2-x-2}$, a solution of -1 or 2 would be extraneous and would not be included in the final solution set.



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Steps to Solve a Rational Equation

1. Find the LCD (lowest common denominator).
2. Multiply every term by the LCD to clear the denominators (use the multiplication property of equality).
3. Solve the resulting equation.
4. Check for Extraneous Solutions (excluded values; solutions that make the denominator equal to 0).
5. State the final solution set.



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Ex) Solve the equation.

$$\frac{x+12}{x^2-3x} = \frac{x+2}{x-3}$$

$x(x-3)$

① excluded values

$$x=0 \quad x-3 \neq 0$$

$$x \neq 0 \quad x \neq 3$$

② LCD = $x(x-3)$

③ $\frac{x+12}{x(x-3)} \cdot \frac{x(x-3)}{1} = \frac{x+2}{x-3} \cdot \frac{x(x-3)}{1}$

④ solve. $\frac{x+12}{-x-12} = \frac{x^2+2x}{-x-12}$

quadratic

standard form

$$0 = x^2 + x - 12$$

$$0 = (x+4)(x-3)$$

zero product property

$$x+4=0 \text{ or } x-3=0$$

$$x = -4$$

~~$x=3$~~
extraneous solution

⑤ check.

solution $x = -4$

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Ex) Solve the equation.

$$\frac{3}{x+2} = \frac{7}{5x-4}$$

(x+2) (5x-4)

① excluded values

$$x+2=0 \quad 5x-4=0$$

$$x \neq -2 \quad x \neq \frac{4}{5}$$

② LCD = $(x+2)(5x-4)$

③

$$\frac{3}{\cancel{x+2}} \cdot \frac{(\cancel{x+2})(5x-4)}{1} = \frac{7}{\cancel{5x-4}} \cdot \frac{(x+2)(\cancel{5x-4})}{1}$$

④ solve. $15x - 12 = 7x + 14$

linear

$$\frac{-7x + 12}{-7x + 12}$$

$$\frac{8x}{8} = \frac{26}{8}$$

$$x = \frac{26}{8} \text{ reduce}$$

⑤ check.

$$x = \frac{13}{4}$$

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Ex) Solve the equation.

$$\frac{9}{4x} - 3 = \frac{13}{12x}$$

$$\frac{9}{4x} - \frac{3}{1} = \frac{13}{12x}$$

$\frac{6 \cdot 2 \cdot x}{3 \cdot 2 \cdot x}$

① excluded values
 $x \neq 0$

② LCD = $2 \cdot 2 \cdot 3 \cdot x = 12x$

③

$$\frac{9}{4x} \cdot \frac{12x}{12x} - \frac{3}{1} \cdot \frac{12x}{12x} = \frac{13}{12x} \cdot \frac{12x}{12x}$$

④ solve

$$\frac{27 - 36x}{-13 + 36x} = \frac{13}{-13 + 36x}$$

linear

$$\frac{14}{36} = \frac{36x}{36}$$

$x = \frac{14}{36}$ reduce

⑤ check.

$x = \frac{7}{18}$

solution

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Ex) Solve the equation.

$$\frac{2}{5x} = \frac{4}{10x}$$

① excluded values $5 \cdot x$ $5 \cdot 2 \cdot x$

$$x \neq 0$$

② $LCD = 5 \cdot 2 \cdot x = 10x$

③ $\frac{2}{\cancel{5x}} \cdot \frac{\overset{\textcircled{2}}{10x}}{1} = \frac{4}{\cancel{10x}} \cdot \frac{10x}{1}$

$4 = 4 \iff$ True statement
or identity statement

\downarrow
infinitely many solutions



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Ex) Solve the equation.

$$\frac{1}{4x} = \frac{3}{7x}$$

$4 \cdot x$ $7 \cdot x$

① excluded value

$$x \neq 0$$

② LCD = $4 \cdot 7 \cdot x = 28x$

③ $\frac{1}{\cancel{4x}} \cdot \frac{\overset{\textcircled{7}}{28x}}{1} = \frac{3}{\cancel{7x}} \cdot \frac{\overset{\textcircled{4}}{28x}}{1}$

$7 \neq 12$ False statement

↓
no solution or \emptyset



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Closure

What is an extraneous solution?

An extraneous solution is a solution that is a value of the variable that makes one or more denominators in the original equation equal to 0.