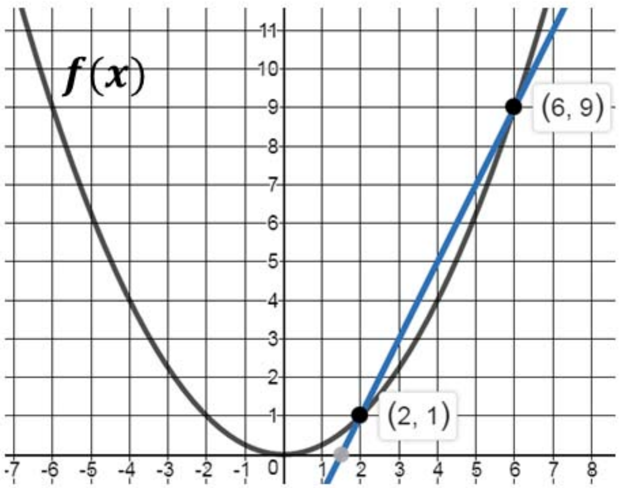
Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Concept

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x} = \frac{\text{change in } f(x) \text{ values}}{\text{change in } x \text{ values}}$$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}; \text{ for the interval } [x_1, x_2]$$

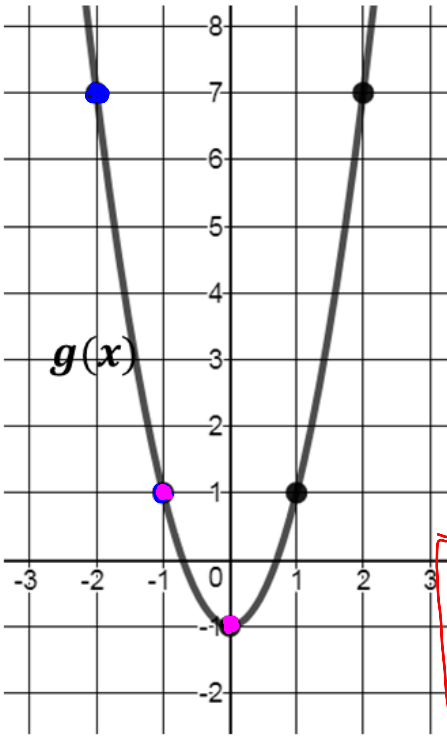


The **average rate of change** of a function for an interval **is equivalent to the slope of the line through the two points of the function**. This line is called the secant line.

For the function $f(x)$ on the interval $[2,6]$ the average rate of change is $\frac{\Delta y}{\Delta x} = \frac{9-1}{6-2} = \frac{8}{4} = 2$

Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Ex) Calculate the average rate of change of the quadratic function shown in the graph for each specified interval.



A) $-2 \le x \le -1$ (x_1, x_2)

① find the points $(-2, 7)$ and $(-1, 1)$

② $\frac{\Delta g(x)}{\Delta x} = \frac{1 - 7}{-1 - (-2)}$ ($-1 + 2$)

$= \frac{-6}{1} = -6$

③ The average rate of change on the interval $-2 \le x \le -1$ is -6.

B) $[-1, 0]$ (x_1, x_2)

① find the points $(-1, 1)$ and $(0, -1)$

② $\frac{\Delta g(x)}{\Delta x} = \frac{-1 - 1}{0 - (-1)}$ ($0 + 1$)

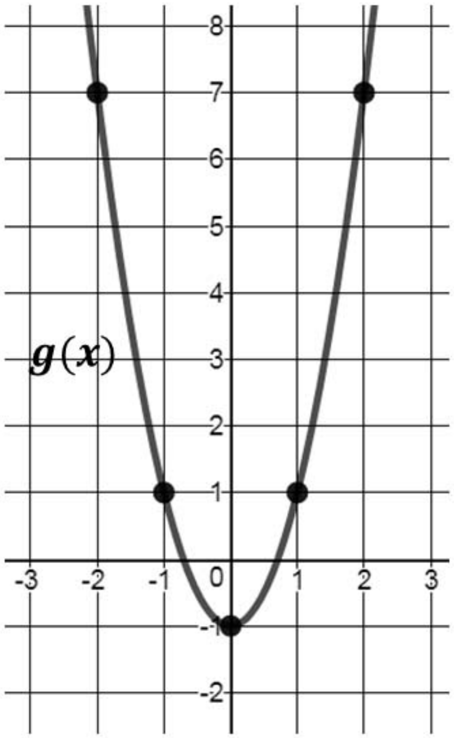
$= \frac{-2}{1} = -2$

③ The average rate of change on the interval $[-1, 0]$ is -2.

C) A negative average rate of change means the function is decreasing on that interval.

Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Practice) Calculate the average rate of change of the quadratic function shown in the graph for each specified interval.



A) [0,2]

1. points: $(0,-1), (2,7)$

2. $\frac{\Delta g(x)}{\Delta x} = \frac{7-(-1)}{2-0} = \frac{8}{2} = 4$

The average rate of change for $g(x)$ on the interval $[0,2]$ is 4.

B) A positive average rate of change means the function is __increasing__ on that interval.

Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Ex) Calculate the average rate of change for $d(x) = 3x^2 - 21x + 2$ on the interval $[-3,2]$.

$x_1 \quad x_2$

① find the points $(-3, 92)$ and $(2, -28)$

$x = -3$ $\quad d(-3) = 3(-3)^2 - 21(-3) + 2$

$3 \cdot 9 \quad -21 \cdot -3$

$= 27 + 63 + 2 = 92$

$x = 2$ $\quad d(2) = 3(2)^2 - 21(2) + 2$

$3 \cdot 4 \quad -21 \cdot 2$

$= 12 + -42 + 2 = -28$

② $\dfrac{\Delta d(x)}{\Delta x} = \dfrac{-28 - 92}{2 - (-3)} = \dfrac{-120}{5} = -24$

$2 + 3$

③ The average rate of change on the interval $[-3, 2]$ is -24.

Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Practice) Calculate the average rate of change for $h(x) = -2x^2 + 17x - 5$ on the interval $[-2,3]$.

1. points: $(-2,-47)$, $(3,28)$

2. $\dfrac{\triangle h(x)}{\triangle x} = \dfrac{28-(-47)}{3-(-2)} = \dfrac{75}{5} = \boxed{15}$

The average rate of change for $h(x)$ on the interval $[-2,3]$ is 15.

Objective: Calculate the average rate of change of a quadratic function for a specified interval.

Practice) Calculate the average rate of change of the quadratic function represented by the table of values over the interval $[-6, -3]$. Round to three decimal places.

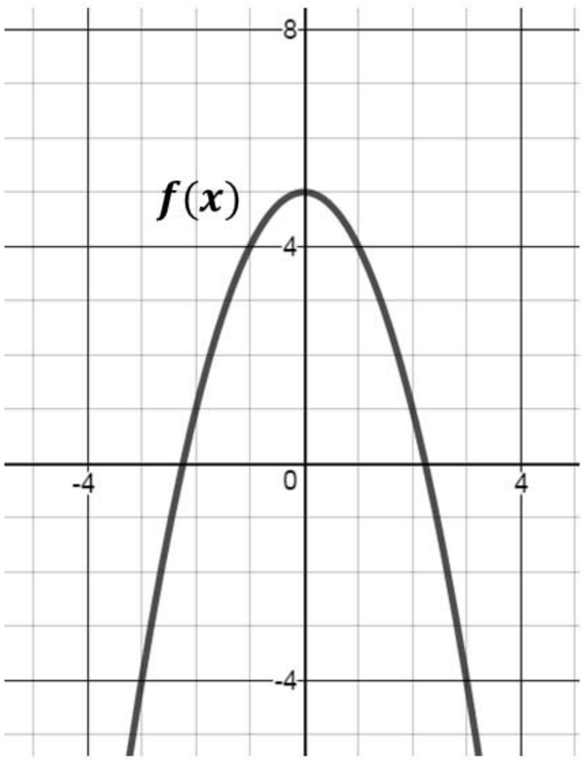
x	$f(x)$
-6	-14
-5	-6
-4	-1
-3	2
-2	3
-1	2
0	-1

1. use points $(-6, -14)$ and $(-3, 2)$
2. $\frac{\Delta f(x)}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$$= \frac{2-(-14)}{-3-(-6)} = \frac{16}{3} \approx 5.333$$

The average rate of change for $f(x)$ on the interval $[-6, -3]$ is about 5.333.

Objective: Calculate the average rate of change of a quadratic function for a specified interval.



Closure

Given the graph of $f(x)$ would you expect the average rate of change for the interval $[4,6]$ to be positive or negative? Explain your reasoning.

I would expect the average rate of change for the interval $[4,6]$ to be negative because the function is decreasing on this interval.