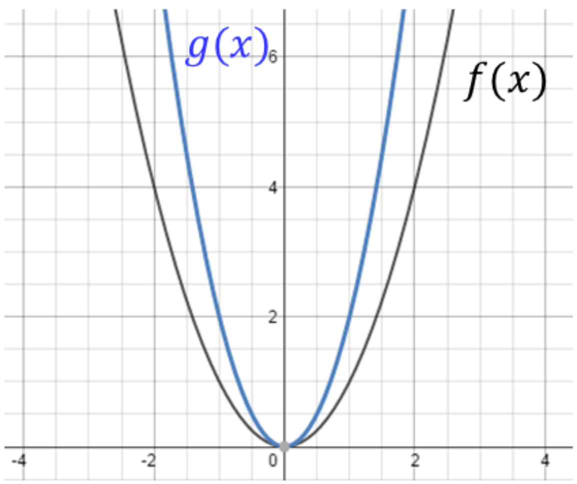
Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

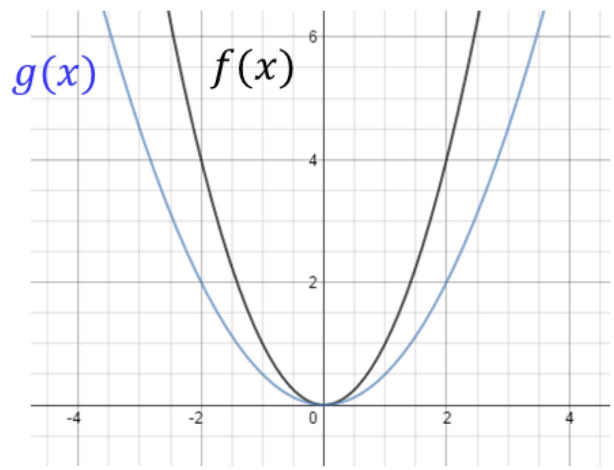
Concept

Value of a	**Transformations of the graph of $f(x)$ to obtain the graph of $g(x) = a \cdot f(x-h) + k$**
$\lvert a \rvert > 1$	**Vertical Stretch** by a factor of $\lvert a \rvert$, and translate h units horizontally and k units vertically.
$\lvert a \rvert < 1$	**Vertical Compression** by a factor of $\lvert a \rvert$, and translate h units horizontally and k units vertically.
$a < 0$	Reflection across the x-axis.

Vertical Stretch: $\lvert a \rvert > 1$



Vertical Compression: $\lvert a \rvert < 1$

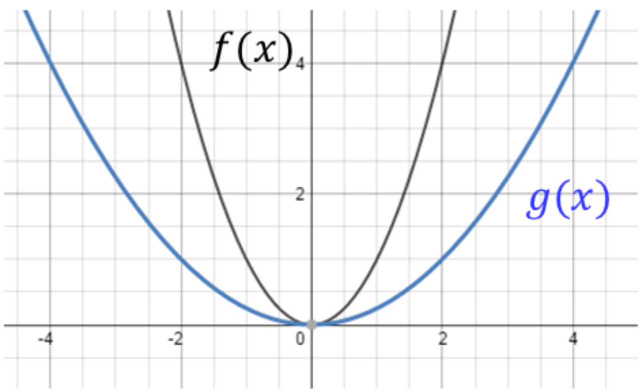


Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

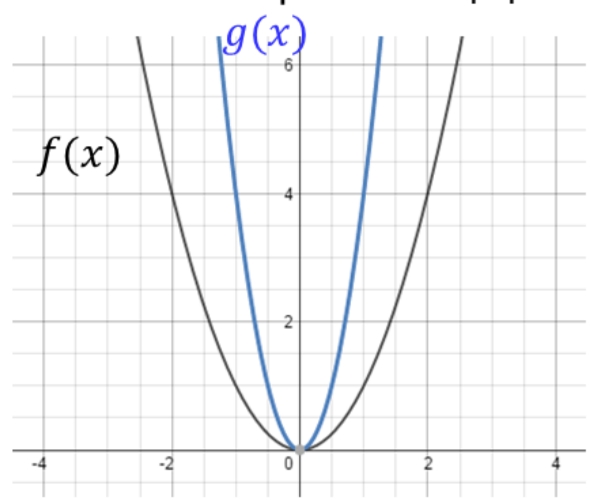
Concept

Value of b reciprocal	**Transformations of the graph of $f(x)$ to obtain the graph of $g(x) = f\left(\frac{1}{b}\cdot(x-h)\right) + k$**
$\lvert b\rvert > 1$	**Horizontal Stretch** by a factor of $\lvert b\rvert$, and translate h units horizontally and k units vertically.
$\lvert b\rvert < 1$	**Horizontal Compression** by a factor of $\lvert b\rvert$, and translate h units horizontally and k units vertically.
$b < 0$	Reflection across the y-axis.

Horizontal Stretch: $\lvert b\rvert > 1$



Horizontal Compression: $\lvert b\rvert < 1$



Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Ex) Given a function, $f(x)$, determine whether the related function $g(x)$ has a vertical stretch or horizontal stretch. Include the factor.

$$g(x) = \overset{a}{3}f(x)$$

$\underset{\text{vert.}}{\underline{a}} = 3 \rightarrow |a| = |3| = 3 \underset{\text{stretch}}{\underline{>1}} \rightarrow$ $\boxed{\text{vertical stretch by a factor of 3}}$

$$g(x) = f\left(\frac{1}{2}(x)\right)$$

$\frac{1}{b} = \frac{1}{2}$

$\star\ \underset{\text{horiz.}}{\underline{b}} = 2 \rightarrow |b| = |2| = 2 \underset{\text{stretch}}{\underline{>1}}$

$\boxed{\text{horizontal stretch by a factor of 2}}$

Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Ex) Given a function, $f(x)$, determine whether the related function $g(x)$ has a vertical compression or horizontal compression. Include the factor.

$g(x) = \overset{a}{\frac{1}{2}} f(x)$

$\underset{\text{vert.}}{\underline{a}} = \frac{1}{2} \rightarrow |a| = \left|\frac{1}{2}\right| = \frac{1}{2} \underset{\text{comp.}}{\underline{\underline{<}}} 1 \rightarrow$ vertical compression by a factor of $\frac{1}{2}$

$g(x) = f(8(x))$

$\frac{1}{b} = 8$

$\star \underset{\text{horiz.}}{\underline{\underline{b}}} = \frac{1}{8} \rightarrow |b| = \left|\frac{1}{8}\right| = \frac{1}{8} \underset{\text{comp.}}{\underline{\underline{<}}} 1$

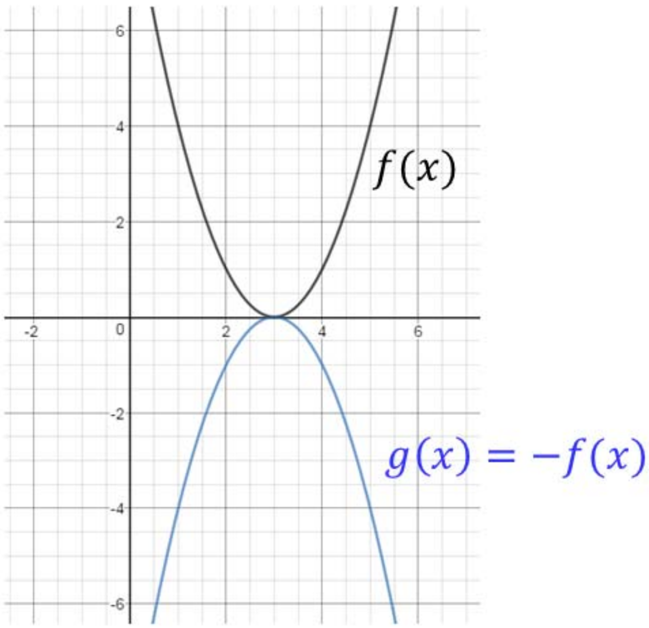
horizontal compression by a factor of $\frac{1}{8}$

Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Concept

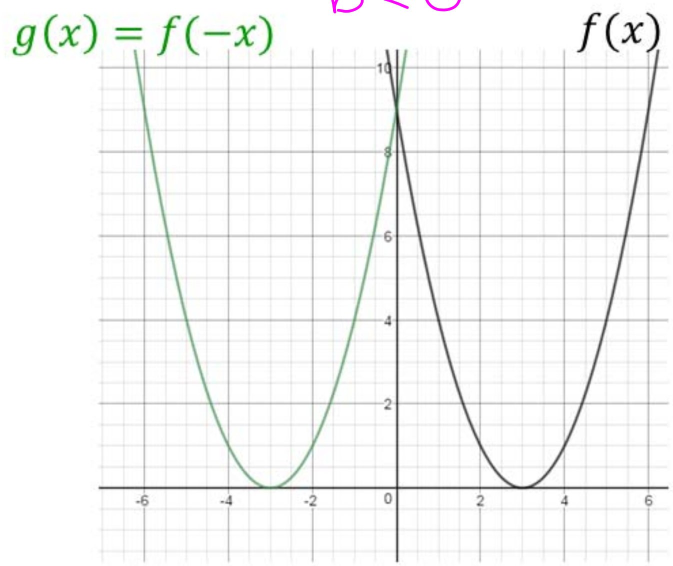
reflection across the x-axis

$a < 0$



reflection across the y-axis

$b < 0$



Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Ex) Given the function $f(x)$ determine whether the related graph of $g(x)$ will have a reflection across the x-axis or y-axis and the type of stretch/compression. Include the factor.

$g(x) = \overset{a}{-2}f(x)$

$a = -2 < 0$ refl.
x-axis

$|a| = |-2| = 2 \geq 1$
vert. stretch

an x-axis reflection and a vertical stretch by a factor of 2

$g(x) = f(-3(x))$

$\frac{1}{b}$

$\frac{1}{b} = -3$

$b = -\frac{1}{3} < 0$ refl.
y-axis

$|b| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$ comp.
horiz.

a y-axis reflection and a horizontal compression by a factor of $\frac{1}{3}$

Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Ex) Describe how to transform the graph of the function $f(x)$ to obtain the graph of the related function $g(x)$.

$$g(x) = f\left(\underbrace{-\tfrac{1}{2}}_{\frac{1}{b}}\left(x \underbrace{- 4}_{\substack{\text{opp}=h \\ h=4}}\right)\right) + 3 \quad k=3$$

$\frac{1}{b} = -\frac{1}{2}$

$b = -2 < 0$ y-axis refl.

$\underline{|b|} = |-2| = 2 \underline{> 1}$

horiz. stretch

$h = 4$ right 4

$k = 3$ up 3

$\boxed{\text{a y-axis reflection, a horizontal stretch by a factor of 2 and a translation right 4 units and up 3 units}}$

$$g(x) = \underbrace{\tfrac{1}{3}}_{a} f\left(x \underbrace{+ 2}_{\substack{\text{opp}=h \\ h=-2}}\right) - 1 \quad k=-1$$

$a = \frac{1}{3} > 0$ no refl.

$|\underline{a}| = \left|\frac{1}{3}\right| = \frac{1}{3} \underline{< 1}$

vert. comp.

$h = -2$ left 2 units

$k = -1$ down 1 unit

$\boxed{\text{a vertical compression by a factor of } \tfrac{1}{3} \text{ and a translation left 2 units and down 1 unit}}$

Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Ex) Describe how to transform the graph of the function $f(x)$ to obtain the graph of the related function $g(x)$.

$$g(x) = f\big(\underbrace{-}_{\frac{1}{b}}(x)\big) \overbrace{-5}^{k}$$

$$g(x) = \underbrace{-3}_{a} f(x \underbrace{-9}_{\text{opp}=h=9})$$

$\frac{1}{b} = -1$ y-axis refl.

$b = -1 < 0$

$|b| = |-1| = 1 = 1$ no stretch no comp.

$k = -5$ down 5

a y-axis reflection and a translation down 5 units

$a = -3 < 0$ x-axis refl.

$|a| = |-3| = 3 \geq 1$ vert. stretch

$h = 9$ right 9

an x-axis reflection, a vertical stretch by a factor of 3 and a translation right 9 units.

Objective: Describe how to transform the graph of $f(x)$ to obtain the graph of the related function $g(x)$.

Closure

Eddie was asked to describe the transformation of the function $f(x)$ to obtain the graph of the related function $g(x) = f\left(-\frac{1}{3}(x)\right) - 5$. Below is his response.

Reflect $f(x)$ across the x-axis, then a horizontal compression by a factor of $\frac{1}{3}$, and translate 5 units left.

Explain how to correct Eddie's response.

To correct Eddie's response I would write: Reflect $f(x)$ across the y-axis, then stretch horizontally by a factor of 3, and translate down 5 units.