Objective: Find rational zeros using the remainder theorem.

Concept

Remainder Theorem: If r is a zero of a polynomial function $f(x)$, then $(x-r)$ is a factor of $f(x)$ and $f(x) \div (x-r)$ has a remainder of 0.

$$\begin{array}{r} p(x) \\ x-r\overline{\smash{)}\,f(x)} \\ \underline{-f(x)} \\ 0 \end{array}$$

It follows that, $f(x) = (x-r) \cdot p(x)$, **where** $p(x)$ **is also a factor of** $f(x)$.

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Steps to find the zeros of a function using the Remainder Theorem

1. Divide the function by the given factor of the function.
2. Write the function as a product of its factors. Don't forget to include the given factor.
3. Factor further, if needed.
4. Use the Zero Product Property
5. Solve for the zeros.

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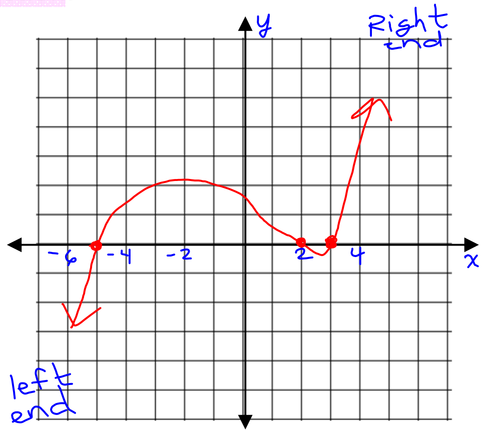
Ex) Given a factor of a polynomial function: a) find the rational zeros (include any multiplicity), b) sketch the graph of the function.

$f(x) = x^3 - 19x + 30$; $(x + 5)$

① divide

$$\begin{array}{rl}
 & \ast\ x^2 - 5x + 6 \\
\ast\ x+5 & \overline{)\ x^3 + 0x^2 - 19x + 30} \\
x^2(x+5) & +(\overset{-}{\cancel{x^3}} \overset{-}{+} 5x^2) \\
 & -5\cancel{x^2} - 19x \\
-5x(x+5) & +(+\cancel{5x^2} + 25x) \\
 & 6x + 30 \\
6(x+5) & +(\overset{-}{\cancel{6x}} \overset{-}{+} 30) \\
 & 0\ \checkmark
\end{array}$$



ⓐ $\boxed{\text{zeros} = -5, 2, 3}$

②* factored form

$f(x) = (x+5)(x^2 - 5x + 6)$

* factor further $(x+5)(x-3)(x-2)$

③ zeros: $0 = (x+5)(x-3)(x-2)$

$$\begin{array}{ccc}
x + 5 = 0 & x - 3 = 0 & x - 2 = 0 \\
-5 \quad -5 & +3 \quad +3 & +2 \quad +2 \\
\hline
x = -5 & x = 3 & x = 2
\end{array}$$

Objective: Find rational zeros using the remainder theorem.

Ex) Given a factor of a polynomial function: a) find the rational zeros (include any multiplicity), b) sketch the graph of the function.

$f(x) = -x^3 + 12x^2 - 45x + 50; (x - 2)$

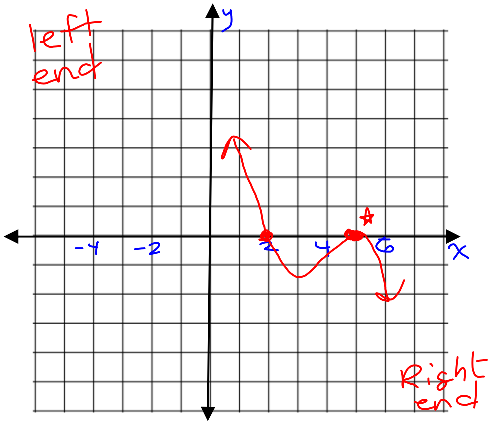
①

$$\begin{array}{r|l} & ☆\,-1x^2 + 10x - 25 \\ \hline ☆\,x-2 & -1x^3 + 12x^2 - 45x + 50 \\ & +(+\cancel{1x^3} \overset{-}{+} 2x^2) \\ \hline & \cancel{10x^2} - 45x \\ & +(-\cancel{10x^2} + 20x) \\ \hline & -\cancel{25x} + 50 \\ & +(+\cancel{25x} \overset{-}{+} 50) \\ \hline & 0 ✓ \end{array}$$

$-1x^2(x-2)$

$10x(x-2)$

$-25(x-2)$



ⓐ $\boxed{\text{zeros: } 2, 5, 5}$

② factored form

$f(x) = ☆(x-2)(-1x^2 + 10x - 25)$

③ ☆ factor further

$☆-1(x^2 - 10x + 25)$

$☆(x-5)(x-5)$

$f(x) = -1(x-2)(x-5)(x-5)$

④ zeros

$-1 \neq 0$

$x - 2 = 0$

$x = 2$

$x - 5 = 0$

$x = 5$

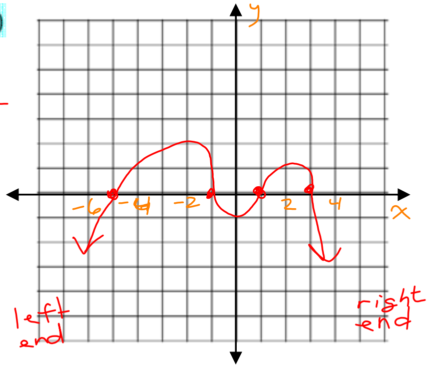
$x - 5 = 0$

$x = 5$

Objective: Find rational zeros using the remainder theorem.

Ex) Given a factor of a polynomial function: a) find the rational zeros (include any multiplicity), b) sketch the graph of the function.

$f(x) = -x^4 - 2x^3 + 16x^2 + 2x - 15; (x - 3)$



①

$^*-1x^3 - 5x^2 + x + 5$

$^*x-3 \overline{)\, -1x^4 - 2x^3 + 16x^2 + 2x - 15}$

$-1x^3(x-3)$ $\quad +(\cancel{+1x^4} \overset{-}{+} 3x^3)$

$-\cancel{5x^3} + 16x^2$

$-5x^2(x-3)$ $\quad +(\cancel{+5x^3} \overset{-}{+} 15x^2)$

$\cancel{1x^2} + 2x$

$x(x-3)$ $\quad +({}^{-}\cancel{x^2} + 3x)$

$5x - 15$

$5(x-3)$ $\quad +({}^{-}5x + 15)$

0 ✓

ⓐ $\boxed{\text{zeros} = -5, -1, 1, 3}$

② * factored form

$f(x) = {}^*(x-3)(-1x^3 - 5x^2 + x + 5)$

$^*-1(x^3 + 5x^2 - x - 5)$

* factor by grouping $\quad x^2(x+5) - 1(x+5)$

$^*(x+5)^*(x^2-1)$

$f(x) = -1(x-3)(x+5)(x^2-1)$

③ zeros $\quad 0 = -1(x-3)(x+5)(x^2-1)$

$-1 \neq 0$

$x - 3 = 0,$
$+3 \quad +3$
$^*\ x = 3$

$x + 5 = 0,$
$-5 \quad -5$
$^*\ x = -5$

$x^2 - 1 = 0$
$+1 \quad +1$
$x^2 = 1$
$\sqrt{\cancel{x^2}} = \pm\sqrt{1}$
$^*\ x = -1, 1$