Objective: Solve a quadratic equation by completing the square

Concept

Solving by Completing the Square includes the procedure to **rewrite a quadratic expression as a perfect square trinomial in its factored form** so that the equation can be solved using the Square Root Property.

Solve by completing the square.

$$x^2 - 18x + 3 = 0$$ standard form

create a perfect square trinomial

$$x^2 - 18x + \_\_\_\_ = -3 + \_\_\_\_$$

$$x^2 - 18x + 81 = -3 + 81$$

$$(x-9)^2 = 78$$

square root property →

$$\sqrt{(x-9)^2} = \pm\sqrt{78} \rightarrow \sqrt{2}\cdot\sqrt{39} \quad \sqrt{2}\cdot\sqrt{3}\cdot\sqrt{13}$$

$$x - 9 = \pm\sqrt{78}$$

$$+9 \quad +9$$

$$\boxed{x = 9 - \sqrt{78},\ 9 + \sqrt{78}}$$

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Steps to Solve by Completing the Square when $a = 1$

1. From standard form, $x^2 + bx + c = 0$, move the constant to write the equation in the form $x^2 + bx = -c$

2. **Create a perfect square trinomial** $x^2 + bx + \left(\frac{b}{2}\right)^2$. Don't forget to balance the equation by adding $\left(\frac{b}{2}\right)^2$ to both sides.

3. Factor the perfect square trinomial and simplify the right side of the equation.

4. Use the square root property. (Don't forget $\pm$.)

5. Finish solving for x. Simplify the solutions as much as possible.

6. Write the final solution set.

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Ex) Solve by completing the square. Give exact solutions in simplest form. State whether the solutions are rational, irrational, or imaginary.

$x^2 - 2x + 18 = 0$ standard form

① $-18 \quad -18$

$x^2 - 2x = -18$

② create a perfect square trinomial

$x^2 - 2x + \underset{\left(\frac{-2}{2}\right)^2 \quad (-1)^2}{1} = -18 + 1$

③ $(x-1)(x-1)$

$(x-1)^2 = -17$

④ square root property $\sqrt{(x-1)^2} = \pm\sqrt{-17}$

$\sqrt{17}\cdot\sqrt{-1}$

$x - 1 = \pm\sqrt{17}\,i$

$+1 \quad +1$

$x = 1 \pm \sqrt{17}\,i$

$x = 1 - \sqrt{17}\,i,\ 1 + \sqrt{17}\,i$

"solutions" imaginary

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$$x^2 - 15 = -14x$$

$+14x \quad +14x$

① standard form

$$x^2 + 14x - 15 = 0$$

$+15 \quad +15$

②

$$x^2 + 14x = 15$$

③ create a perfect square trinomial

$$x^2 + 14x + \underbrace{49}_{\left(\frac{14}{2}\right)^2 = (7)^2} = 15 + 49$$

$15 + 49 = 64$

④

$$(x+7)(x+7)$$

$$(x+7)^2 = 64$$

⑤ square root prop.

$$\sqrt{(x+7)^2} = \pm\sqrt{64}$$

$$x + 7 = \pm 8$$

$$x + 7 = -8, \ 8$$

$-7 \quad -7 \quad -7$

$$x = -15, \ 1$$

rational