Objective: Define and Evaluate Logarithms

Concept

An exponential function such as $f(x) = 2^x$ **accepts values of the exponent as inputs** and **delivers the corresponding value of the power as the outputs.**

The inverse of an exponential function is called a logarithmic function.

The inputs (x-values/domain) of a logarithmic function represent the value of a power.

The corresponding outputs (y-values/range) represent the value of the exponent.

For $f(x) = 2^x$, the inverse function is $f(x) = log_2 x$, which is read either as "the logarithm with base 2 of x" or simply as "log base 2 of x."

A key concept to remember:

A logarithm is equal to an exponent value.

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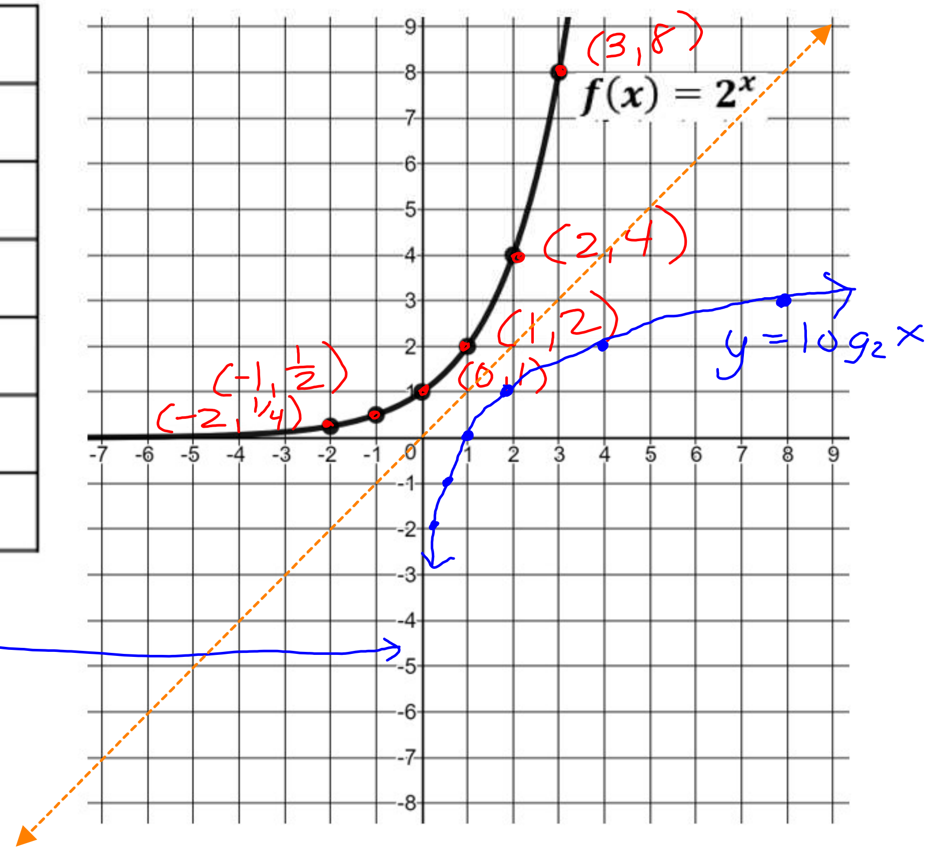
Concept

exponents

The graph of the function $f(x) = 2^x$ is shown. Graph its inverse $f(x) = log_2 x$ and then complete the table.

x	$f(x) = \log_2 x$
8	3
4	2
2	1
1	0
1/2	-1
1/4	-2



asymptote of $y = \log_2 x$

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In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = log_b\, x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = log_3\, x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = log_{\frac{1}{4}}\, x$.

The inverse relationship between exponential functions and logarithmic functions also means that you can write an exponential equation of the form $b^x = a$ as a logarithmic equation of the form $log_b a = x$.

Exponential Form | **Logarithmic Form**

$$b^x = a \qquad\qquad log_b\, a = x$$

Where the **base** $b > 0$, $b \neq 1$ and the **argument** $a > 0$

base exponent argument

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Ex) Complete the table by writing each given equation in its alternate form.

Exponential Form	Logarithmic Form
$4^3 = 64$ (base: 4; exp.: 3; argument: 64)	$\log_4 64 = 3$
$81^{1/4} = 3$	$\log_{81} 3 = \frac{1}{4}$ (base: 81; exp.: $\frac{1}{4}$)
$\left(\frac{2}{3}\right)^p = q$ (base: $\frac{2}{3}$; exp.: p; argument: q)	$\log_{\frac{2}{3}} q = p$
$10^4 = 10{,}000$	$\log 10{,}000 = 4$ means: $\log_{10} 10{,}000 = 4$ (exp.: 4)
$5^{-2} = \frac{1}{25}$ (exp.: -2)	$\log_5 \frac{1}{25} = -2$
$e^2 \approx 7.389$	$\log_e 7.389 \approx 2$

$\ln 7.389 \approx 2$

$Ln\ 7.389 \approx 2$

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Some logarithms can be evaluated without a calculator. If it is possible to write the exponential form with the powers on both sides with the same base, then we can use the Exponential Property of Equality to find the logarithm's value.

Exponential Property of Equality

If $b^m = b^n$ same base

Then $m = n$ equal exponents

Steps to Evaluate a Logarithm Without a Calculator

1. Rewrite the logarithm in exponential form. Use a variable to represent the logarithm's value. (the exponent)
2. Rewrite both sides of the equation so they are powers of the same base.
3. Use the Exponential Property of Equality and then solve for the variable if necessary. Make sure you understand the variable's value is the value of the logarithm and write your conclusion.

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Ex) Evaluate each logarithm without a calculator.

$log_4 16$

exponent

① $\log_4 16 = y$

② exponential form

$4^y = 16$

③ get the same base

$4^y = 4^2$

④ use exponential property of equality

$y = 2$

⑤ conclusion

$\boxed{\log_4 16 = 2}$

$log_5\left(\frac{1}{125}\right)$

exponent

① $\log_5 \frac{1}{125} = y$

② $5^y = \frac{1}{125}$

③ $5^y = \frac{1}{5^3}$

$5^y = 5^{-3}$

④ $y = -3$

⑤ $\boxed{\log_5 \frac{1}{125} = -3}$

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Ex) Evaluate each logarithm without a calculator.

$log_{36}6$

1. $\log_{36} 6 = y$
2. $36^y = 6$
3. $(6^2)^y = 6$
   $6^{2y} = 6^1$
4. $2y = 1$
   $y = \frac{1}{2}$
5. $\boxed{\log_{36} 6 = \frac{1}{2}}$

$log_{\frac{1}{7}}49$

1. $\log_{\frac{1}{7}} 49 = y$
2. $\left(\frac{1}{7}\right)^y = 49$
3. $(7^{-1})^y = 7^2$
   $7^{-1y} = 7^2$
4. $-1y = 2$
   $y = -2$
5. $\boxed{\log_{\frac{1}{7}} 49 = -2}$

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Ex) Evaluate each logarithm without a calculator.

$$log_4 32$$

1. $\log_4 32 = y$
2. $4^y = 32$
3. $(2^2)^y = 2^5$

   $2^{2y} = 2^5$
4. $2y = 5$

   $y = \frac{5}{2}$
5. $\boxed{\log_4 32 = 5/2}$

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Ex) If $f(x) = log_{\frac{1}{2}}x$, find the following without a calculator.

$f(4)$ (x)

evaluate

① $f(4) = \log_{\frac{1}{2}} 4$

② $y = \log_{\frac{1}{2}} 4$

③ $\left(\frac{1}{2}\right)^y = 4$

$(2^{-1})^y = 2^2$

$2^{-1y} = 2^2$

④ $-1y = 2$

$y = -2$

⑤ $\boxed{f(4) = -2}$

$f(2\sqrt{2})$ (x)

evaluate

① $f(2\sqrt{2}) = \log_{\frac{1}{2}} 2\sqrt{2}$

② $y = \log_{\frac{1}{2}} 2\sqrt{2}$

③ $\left(\frac{1}{2}\right)^y = 2\sqrt{2}$

$(2^{-1})^y = 2^1 \cdot 2^{1/2}$ ($1 + 1/2$)

$2^{-1y} = 2^{1\frac{1}{2}}$

$2^{-1y} = 2^{3/2}$

④ $-1y = 3/2$

$y = -3/2$

⑤ $\boxed{f(2\sqrt{2}) = \frac{-3}{2}}$