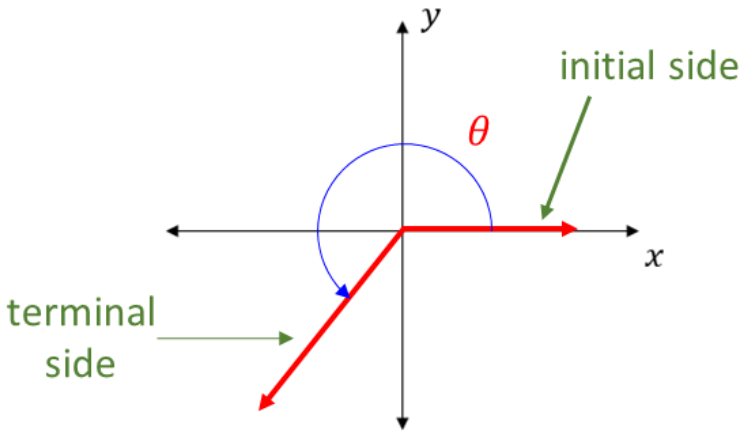
Objective: Understand Degree and Radian Measure of Angles

## Concept

You know from earlier learning that an angle of $180°$ makes a straight line. It is also **possible to have angles that measure greater than $180°$ and angles that have negative measures**.

An **angle in standard position**, or **a central angle**, has its **vertex at the origin** and its **initial side on the positive $x$-axis**. The **terminal side of the angle can be in any quadrant or on an axis**.



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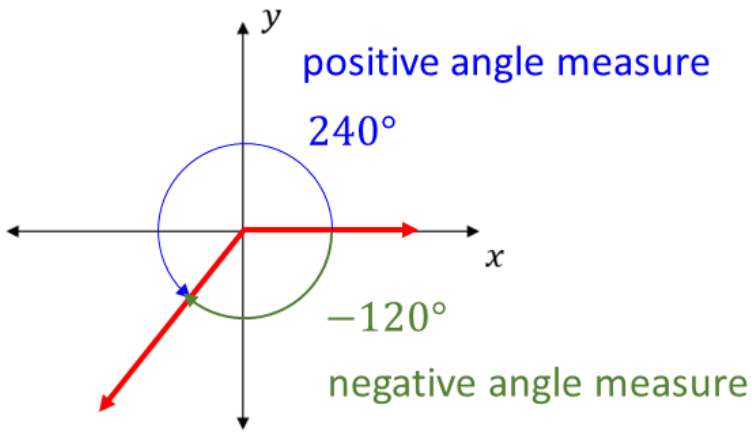
A **positive angle** measure is **measured in a counterclockwise direction** from the positive $x$-axis.

A **negative angle** measure is **measured in a clockwise direction** from the positive $x$-axis.



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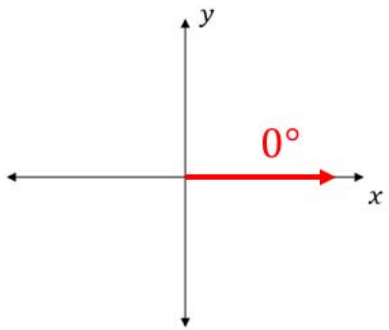
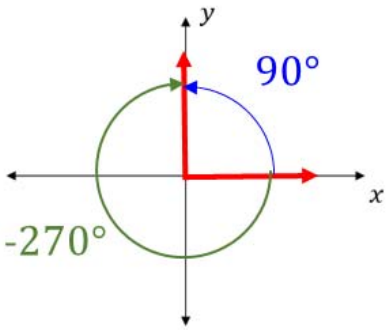
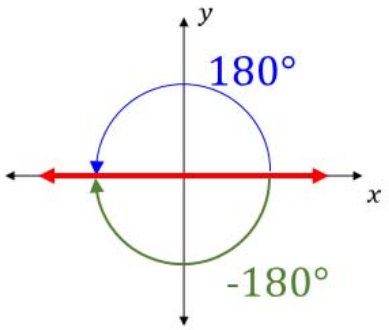
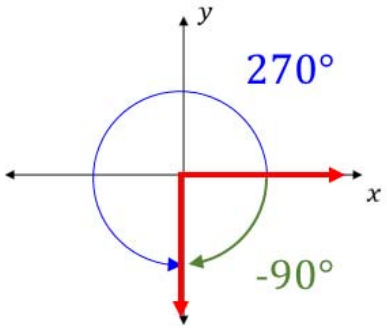
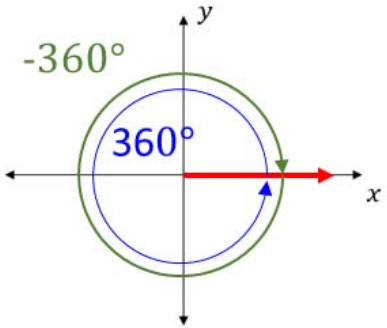
A **quadrantal angle** is **an angle in standard position with its terminal side on an axis**. Examples of quadrantal angles are:
$0°, 90°, 180°, 270°, 360°, -90°, -180°, -360°$



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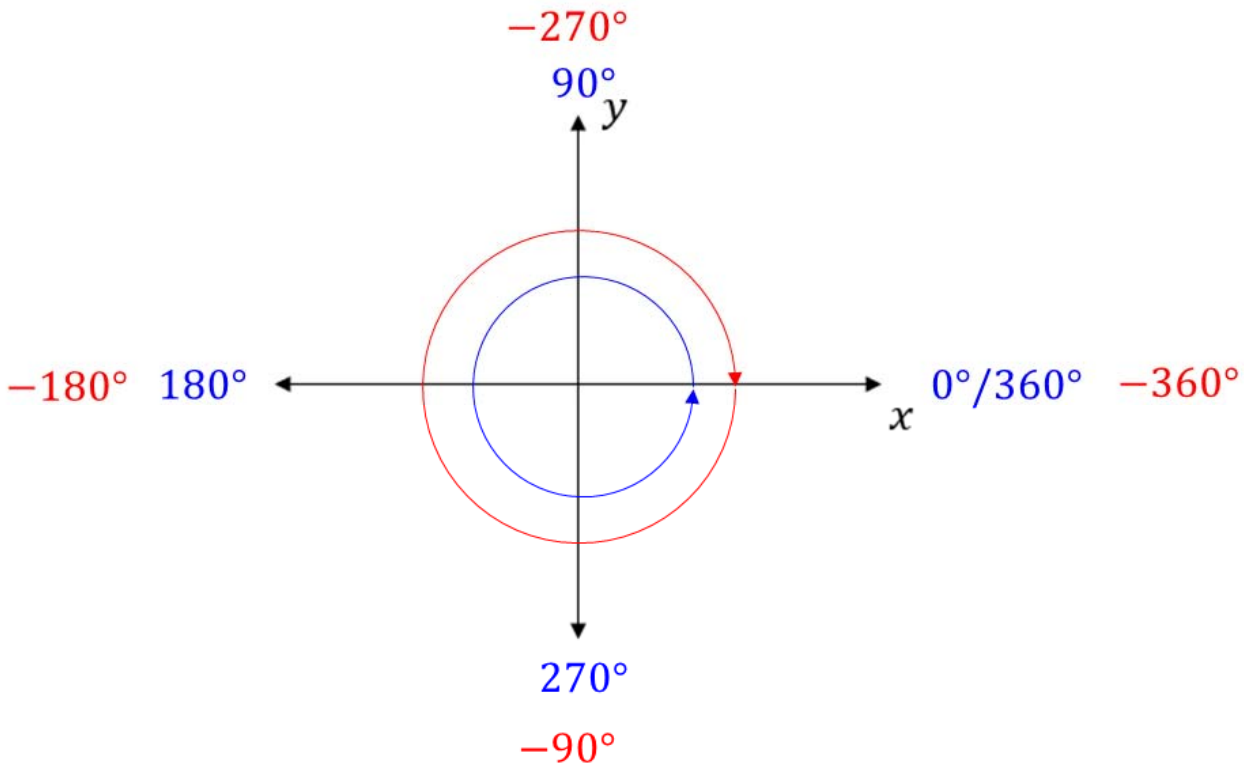
## Concept

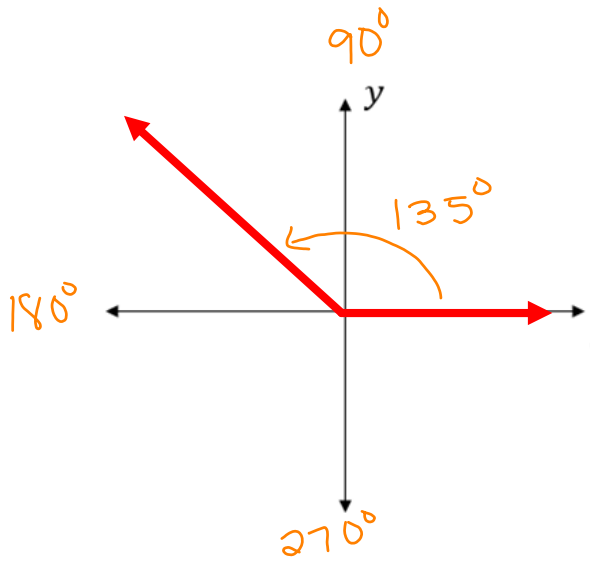
**First Revolution Positive and Negative Quadrantal Angles in Degree Measure**



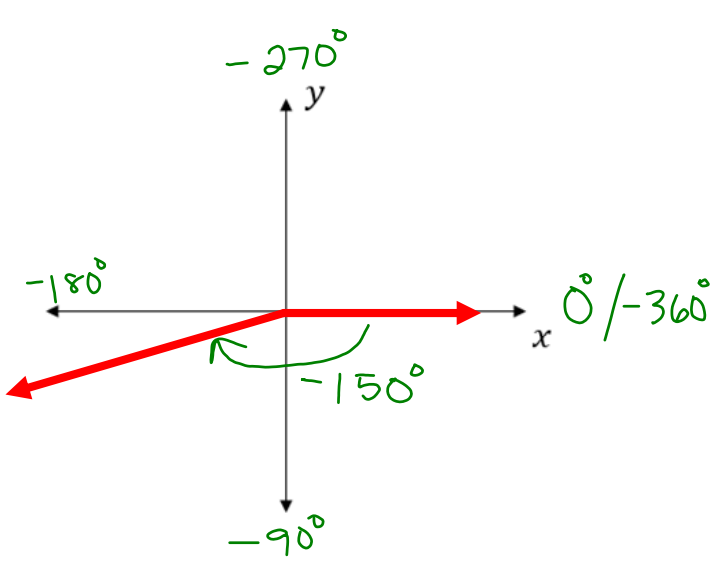
## Objective: Understand Degree and Radian Measure of Angles

Ex) Draw each angle in standard position.

$135°$



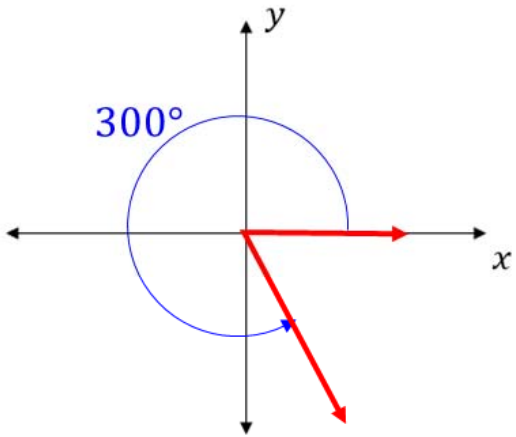

$-150°$



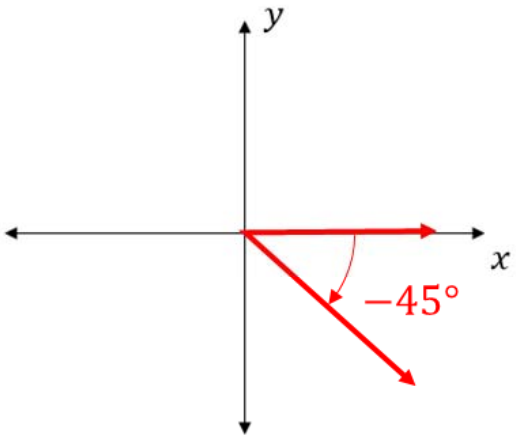
## Objective: Understand Degree and Radian Measure of Angles

Practice) Draw each angle in standard position.

$300°$



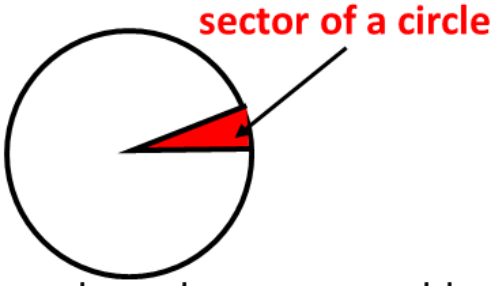

$-45°$



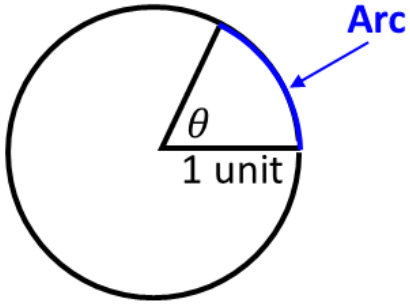
Objective: Understand Degree and Radian Measure of Angles

## Concept

The concept of degrees is based on dividing a circle into 360 equal sectors. Each sector is equal to 1°.




Since a degree measure is not a real number, some problems cannot be solved using degrees. **Another way to measure angles is in radians. The radian measure of an angle is a real number measure that is equal to the length of the arc subtended by the angle in a circle with a radius of 1 unit.**



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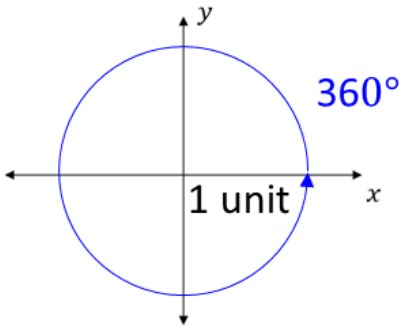
## Concept

In a circle of radius 1 unit, the circumference is equal to $2\pi$. The circumference is the arc subtended by an angle of $360°$. Therefore, $\mathbf{360° = 2\pi}$ **radians**.

$$C = 2\pi r$$
$$C = 2\pi \cdot 1$$
$$C = 2\pi$$




$360° = 2\pi$ radians

The equivalency ratio $\frac{2\pi\ radians}{360°}$, which can be reduced to $\frac{\pi\ radians}{180°}$, can be used to convert degree measure to radian measure.

Its reciprocal, $\frac{180°}{\pi\ radians}$, can be used to convert radians to degrees.

Objective: Understand Degree and Radian Measure of Angles

Concept

| | |
|---|---|
| Degrees to Radians Conversion Ratio | $\frac{\pi\ radians}{180°}$ |
| Radians to Degrees Conversion Ratio | $\frac{180°}{\pi\ radians}$ |

Objective: Understand Degree and Radian Measure of Angles

Ex) Convert each degree measure to radians and each radian measure to degrees.

| Degree Measure | Radian Measure |
|---|---|
| $135°$ | $\frac{135^\circ}{1} \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{135\pi}{180} = \frac{3\pi}{4}$ |
| $570°$ | $\frac{570^\circ}{1} \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{570\pi}{180} = \frac{19\pi}{6}$ |
| $\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ$ | $\frac{5\pi}{6}$ |
| $-30°$ | $\frac{-30^\circ}{1} \cdot \frac{\pi \text{ rad}}{180^\circ} = -\frac{\pi}{6}$ |
| $\frac{9\pi \text{ rad}}{4} \cdot \frac{180^\circ}{\pi \text{ rad}} = 405^\circ$ | $\frac{9\pi}{4}$ |
| $\frac{\pi \text{ rad}}{3} \cdot \frac{180^\circ}{\pi \text{ rad}} = 60^\circ$ | $\frac{\pi}{3}$ |

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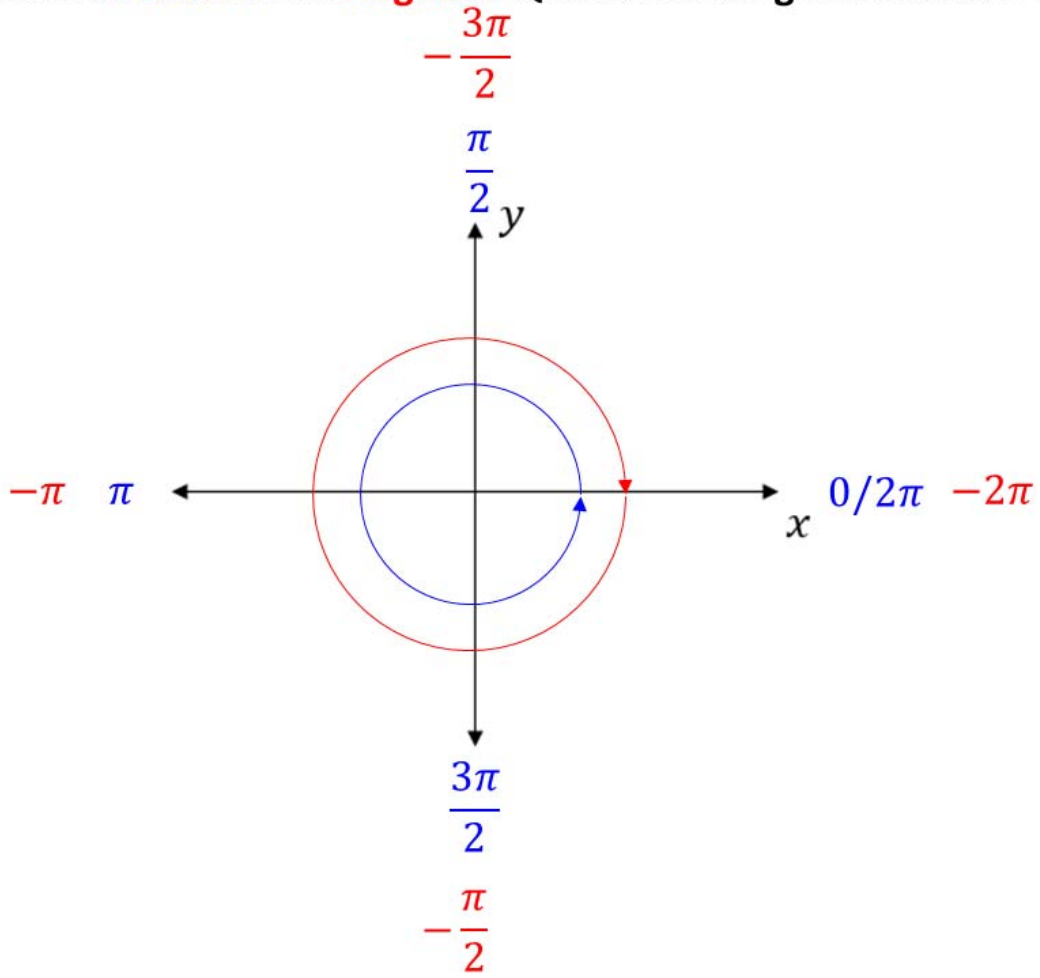
Concept

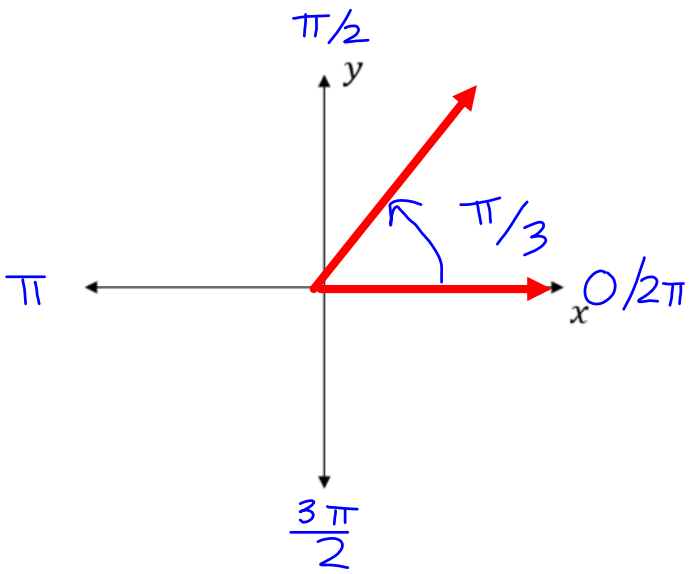
**First Revolution Positive and Negative Quadrantal Angles in Radian Measure**



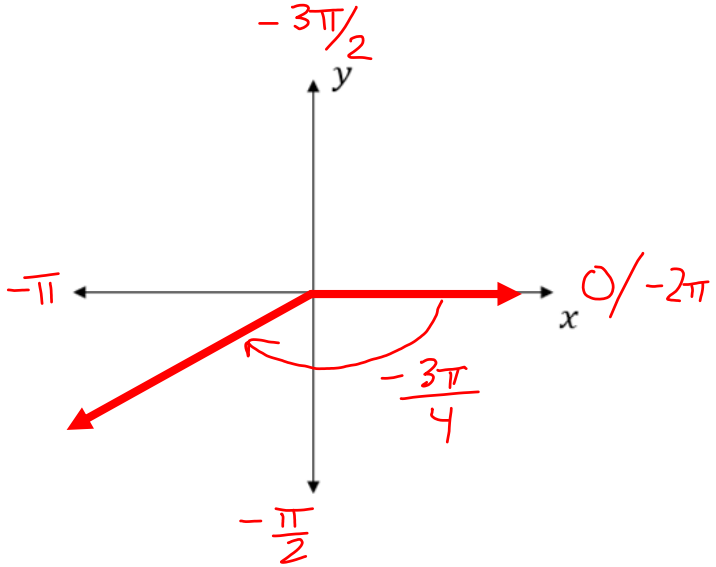
## Objective: Understand Degree and Radian Measure of Angles

Ex) Draw each angle in standard position.

$\frac{\pi}{3}$




$-\frac{3\pi}{4}$



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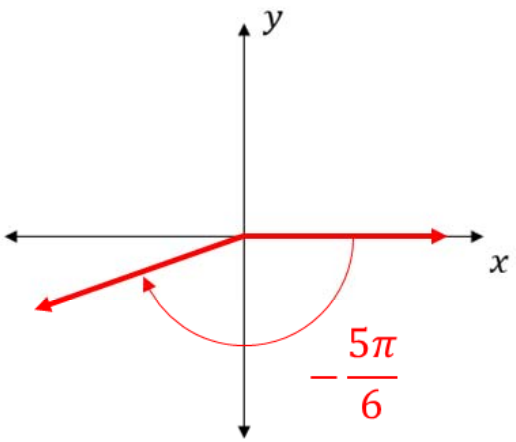
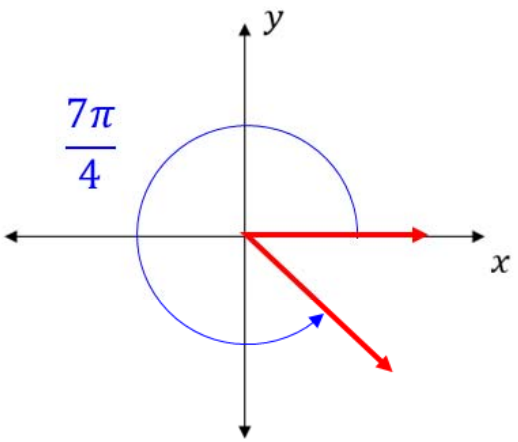
Practice) Draw each angle in standard position.

$-\frac{5\pi}{6}$




$\frac{7\pi}{4}$



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## Closure

What is the difference between a $90°$ angle of rotation and a $-90°$ angle of rotation?

The $90°$ angle has its terminal side along the positive y-axis. The $-90°$ has its terminal side along the negative y-axis.