

Objective: Simplify Sums and Differences of Radical Expressions

Concept

When adding and subtracting radical expressions, only like terms can be combined.

Like terms have the same index and radicand.

Examples of Like Terms

$$3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$$

$$4\sqrt[3]{5} + 9\sqrt[3]{5} = 13\sqrt[3]{5}$$

Examples of Unlike Terms

$$7\sqrt{2} + 4\sqrt{3}$$

$$-2\sqrt{6} + 8\sqrt[3]{6}$$

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Concept

Radical expressions are in simplest form when certain conditions are met.

- 1. The radicand of an nth root has no factors of an nth power.**

Simplified

$$2\sqrt{3}$$

Not Simplified

$$\sqrt{12}$$

- 2. The radicand is not a fraction or decimal.**

Simplified

$$\frac{\sqrt{3}}{3}$$

Not Simplified

$$\sqrt{\frac{1}{3}}$$

- 3. There are no radical expressions in the denominator of a ratio.**

Simplified

$$\frac{2\sqrt{5}}{5}$$

Not Simplified

$$\frac{2}{\sqrt{5}}$$

- 4. Like terms have been combined.**

Simplified

$$4\sqrt{2} + 3\sqrt{2} + 5\sqrt{3} \\ = 7\sqrt{2} + 5\sqrt{3}$$

Not Simplified

$$4\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$$



The first ten perfect cubes

$$\begin{array}{ccccc} 1 & 8 & 27 & 64 & 125 \\ = 1^3 & = 2^3 & = 3^3 & = 4^3 & = 5^3 \end{array}$$

$$\begin{array}{ccccc} 216 & 343 & 512 & 729 & 1000 \\ = 6^3 & = 7^3 & = 8^3 & = 9^3 & = 10^3 \end{array}$$

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Ex) Simplify each expression.

$$3\sqrt{32}$$

$$\begin{aligned} \textcircled{1} \quad & 3 \cdot \sqrt{4} \cdot \sqrt{8} \\ & 3 \cdot 2 \cdot \sqrt{8} \\ & 6\sqrt{8} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & 6 \cdot \sqrt{4} \cdot \sqrt{2} \\ & 6 \cdot 2 \cdot \sqrt{2} \\ & \boxed{12\sqrt{2}} \end{aligned}$$

or

$$\begin{aligned} \textcircled{1} \quad & 3\sqrt{16} \cdot \sqrt{2} \\ & 3 \cdot 4 \cdot \sqrt{2} \\ & \boxed{12\sqrt{2}} \end{aligned}$$

$$4\sqrt[3]{250}$$

$$\begin{aligned} \textcircled{1} \quad & 4 \cdot \sqrt[3]{125} \cdot \sqrt[3]{2} \\ & 4 \cdot 5 \cdot \sqrt[3]{2} \\ & \boxed{20\sqrt[3]{2}} \end{aligned}$$

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Ex) Simplify each expression.

$$\sqrt{81} + 6\sqrt{2} - 11\sqrt{2}$$

① simplify

$$9 + 6\sqrt{2} - 11\sqrt{2}$$

② add like terms

$$9 - 5\sqrt{2}$$

$$\sqrt{5} + 2\sqrt{6} - \sqrt{6} + \sqrt{5}$$

① add like terms

$$2\sqrt{5} + \sqrt{6}$$

Objective: Simplify Sums and Differences of Radical Expressions

Ex) Simplify each expression.

$$\underline{-5\sqrt{24}} - \underline{5\sqrt{6}} + \underline{2\sqrt{54}}$$

① simplify

$$\begin{array}{l} -5\sqrt{4 \cdot 6} \quad \downarrow \quad + 2 \cdot \sqrt{9 \cdot 6} \\ -5 \cdot 2 \cdot \sqrt{6} \quad \downarrow \quad + 2 \cdot 3 \cdot \sqrt{6} \\ -10 \cdot \sqrt{6} - 5\sqrt{6} + 6\sqrt{6} \end{array}$$

② add like terms

$$\boxed{-9\sqrt{6}}$$

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Ex) Simplify each expression.

$$\begin{aligned} & \sqrt[3]{192} - 3\sqrt[3]{81} \\ & \sqrt[3]{64} \cdot \sqrt[3]{3} - 3 \cdot \sqrt[3]{27} \cdot \sqrt[3]{3} \\ & 4\sqrt[3]{3} - 3 \cdot 3 \cdot \sqrt[3]{3} \\ & 4\sqrt[3]{3} - 9\sqrt[3]{3} \\ & \boxed{-5\sqrt[3]{3}} \end{aligned}$$

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Ex) Simplify each expression.

$$\underline{5\sqrt[3]{32}} - \underline{\sqrt[3]{375}} + \underline{\sqrt[3]{108}}$$

$$5 \cdot \sqrt[3]{8} \cdot \sqrt[3]{4} - \sqrt[3]{125} \cdot \sqrt[3]{3} + \sqrt[3]{27} \cdot \sqrt[3]{4}$$

$$5 \cdot 2 \cdot \sqrt[3]{4}$$

$$\underline{10\sqrt[3]{4}} - 5\sqrt[3]{3} + \underline{3\sqrt[3]{4}}$$

$$\boxed{-5\sqrt[3]{3} + 13\sqrt[3]{4}}$$

Objective: Simplify Sums and Differences of Radical ExpressionsClosure

Becky simplified the following radical expression. Do you agree or disagree with her answer? Why?

Becky's Work:

$$\begin{aligned} & \sqrt[3]{24} + 2\sqrt{12} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{3} + 2 \cdot \sqrt{4} \cdot \sqrt{3} \\ &= 2 \cdot \sqrt[3]{3} + 2 \cdot 2 \cdot \sqrt{3} \\ &= 2\sqrt[3]{3} + 4\sqrt{3} \\ &= 6\sqrt[3]{3} \end{aligned}$$

Becky's work is incorrect because she combined a square root term and a cubic root term, which are not like terms, in the last step. The correct answer is $2\sqrt[3]{3} + 4\sqrt{3}$.