Objective: Find the restricted domain of a square root function.

Concept

The domain of a square root function is determined by making the radicand non-negative $(radicand \geq 0)$ **and solving for** x**.** This is necessary so the range will be part of the set of all real numbers instead of the set of complex numbers.

Objective: Find the restricted domain of a square root function.

Ex) Find the domain of the square root function without using a graph. Use set and interval notation.

$$g(x) = 2\sqrt{\underbrace{5x-3}_{\text{radicand}}} - 2$$

① radicand ≥ 0

$5x - 3 \geq 0$

$+3 \quad +3$

$5x \geq 3$

$\frac{5x}{5} \geq \frac{3}{5}$

$x \geq \frac{3}{5}$

② set notation

$\{x \mid x \geq 3/5\}$

interval

$\left[\frac{3}{5}, \infty\right)$

domain

Objective: Find the restricted domain of a square root function.

Ex) Find the domain of the square root function without using a graph. Use set and interval notation.

$$g(x) = \sqrt{-\frac{1}{2}(x+2)} + 1$$

① radicand ≥ 0

$$-2 \cdot -\frac{1}{2}(x+2) \geq 0 \quad \div -2$$

$$x + 2 \leq 0$$

$$-2 \qquad -2$$

$$x \leq -2$$

② domain

$\{x \mid x \leq -2\}$

set notation

$(-\infty, -2]$

interval notation

Objective: Find the restricted domain of a square root function.

Ex) Find the domain of the square root function without using a graph. Use set and interval notation.

$$g(x) = \sqrt{x^2 - 12} + 10$$

① radicand ≥ 0

$x^2 - 12 \geq 0$

ⓐ $x^2 - 12 = 0$

$+12 \quad +12$

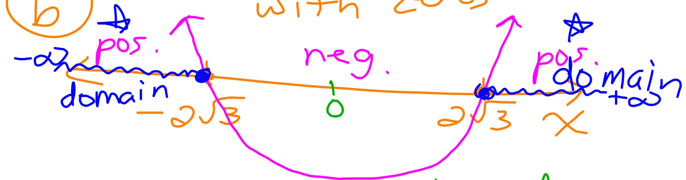
$x^2 = 12$

$\sqrt{\cancel{x^2}} = \pm\sqrt{12}$

$\sqrt{4} \cdot \sqrt{3}$

$x = -2\sqrt{3}, 2\sqrt{3}$

ⓑ create number line with zeros



ⓒ test the intervals using $x = 0$ and $x^2 - 12$

② domain

$\{x \mid x \leq -2\sqrt{3} \text{ or } x \geq 2\sqrt{3}\}$

set notation

$(-\infty, -2\sqrt{3}] \cup [2\sqrt{3}, \infty)$

interval notation

Objective: Find the restricted domain of a square root function.

Ex) Find the domain of the square root function without using a graph. Use set and interval notation.

$$g(x) = \sqrt{49 - x^2} - 1$$

① $49 - x^2 \geq 0$

ⓐ $49 - x^2 = 0$

$(7)^2 - (x)^2$

$(7 - x)(7 + x) = 0$

$7 - x = 0$ or $7 + x = 0$

$+x \quad +x \qquad -7 \quad -7$

$7 = x \qquad x = -7$

ⓑ create number line using zeros of $49 - x^2 = 0$

neg   pos   neg x

-7 domain 7

ⓒ test intervals using $x = 0$ and $49 - x^2$

② domain

$\{x \mid -7 \leq x \leq 7\}$

set notation

$[-7, 7]$

interval notation

Objective: Find the restricted domain of a square root function.

Ex) Find the domain of the square root function without using a graph. Use set and interval notation.

$$g(x) = \sqrt{x^2 - 5x + 4} - 1$$

① $x^2 - 5x + 4 \geq 0$

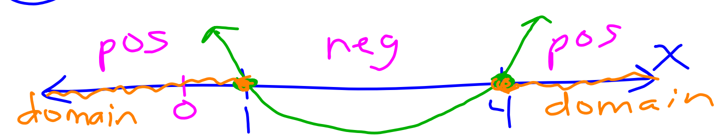
ⓐ $x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0$ $\quad$ $x - 1 = 0$

$x = 4$ $\quad$ $x = 1$

ⓑ create number line



ⓒ test intervals using $x = 0$ and $x^2 - 5x + 4$

② domain

$\{x \mid x \leq 1 \text{ or } x \geq 4\}$

set notation

$(-\infty, 1] \cup [4, \infty)$

interval notation

Objective: Find the restricted domain of a square root function.

Closure

Adam found the restricted domain of a square root function. His work is shown. Do you agree or disagree with his conclusion? Explain your reasoning.

$$g(x) = \sqrt{-\frac{1}{3}(x-5)} - 19$$

$$-\frac{1}{3}(x-5) \geq 0$$

$$-3 \cdot -\frac{1}{2}(x-5) \geq 0 \cdot -3$$

$$x - 5 \leq 0$$

$$+5 \quad +5$$

$$x \leq 5$$

Domain: $\{x | x \leq 5\}$; $(-\infty, 5]$

I agree with Adam's conclusion because he started by setting the radicand greater than or equal to zero, then he solved for x, correctly changing the inequality sign when he multiplied by a negative number, in this case -3, before adding 5 to both sides to end with the restricted domain.