Objective: Graph and Identify Key Features of Exponential Growth Functions

Concept

An exponential function is a function of the form $f(x) = a(b)^{c(x-h)} + k$, where $b > 0$ and $b \neq 1$. The domain of every exponential function is all real numbers because the value of the exponent can be any real number.

There is no single parent exponential function because each choice of the base b determines a different function. For example, $f(x) = 2^x$ and $f(x) = 10^x$ are both parent functions for their respective bases.

What to include in the graph of an exponential function.

- **horizontal asymptote;** Note: for the function $f(x) = a(b)^{cx-h} + k$, the horizontal asymptote is $y = k$.
- **key points**, including the y**-intercept** and/or **zero** when reasonable
- **end behavior**

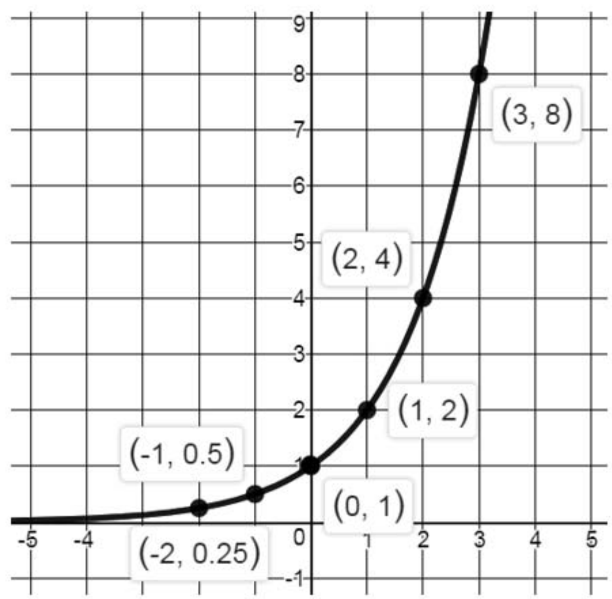
Objective: Graph and Identify Key Features of Exponential Growth Functions

Concept

The function $\boldsymbol{f(x) = 2^x}$ is the parent function for all exponential functions with base 2. These functions have the form $f(x) = a \cdot 2^{\frac{1}{b}(x-h)} + k$.

x	$f(x) = 2^x$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

$\boldsymbol{f(x) = 2^x}$



Domain: the set of all real numbers; $\{x|-\infty < x < +\infty\}$; $(-\infty, +\infty)$
Range: $\{y|y > 0\}$; $(0, +\infty)$
Horizontal Asymptote: $y = 0$
End Behavior: $as\ x \to -\infty, f(x) \to 0; as\ x \to +\infty, f(x) \to +\infty$

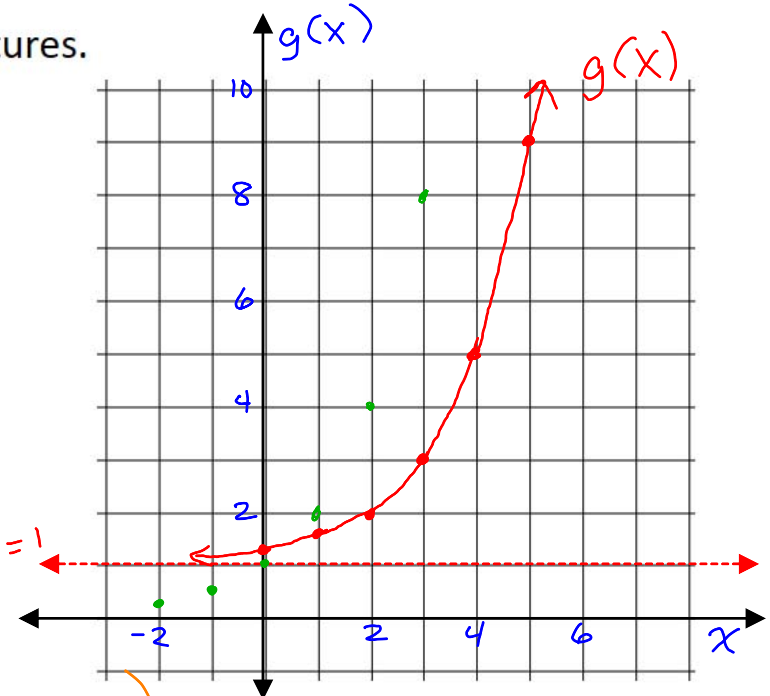
Objective: Graph and Identify Key Features of Exponential Growth Functions

Ex) Graph the function. State the key features.

$g(x) = 2^{x-2} + 1$ (opp = h) k

a = 1, h = 2 (right), k = 1 (up)

y-int. → $g(0) = 2^{0-2} + 1$
$= 2^{-2} + 1$
$= \frac{1}{4} + 1 = 1\frac{1}{4}$



Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y > 1\}$ / $(1, \infty)$
(set notation) (interval notation)

y-intercept: $(0, 1\frac{1}{4})$

Horizontal Asymptote: $y = 1$ ($y = k$)

End Behavior: as $x \to -\infty$, $g(x) \to 1$
as $x \to \infty$, $g(x) \to \infty$

Objective: Graph and Identify Key Features of Exponential Growth Functions

Ex) Graph the function. State the key features.

$$g(x) = -2^{x+3}$$

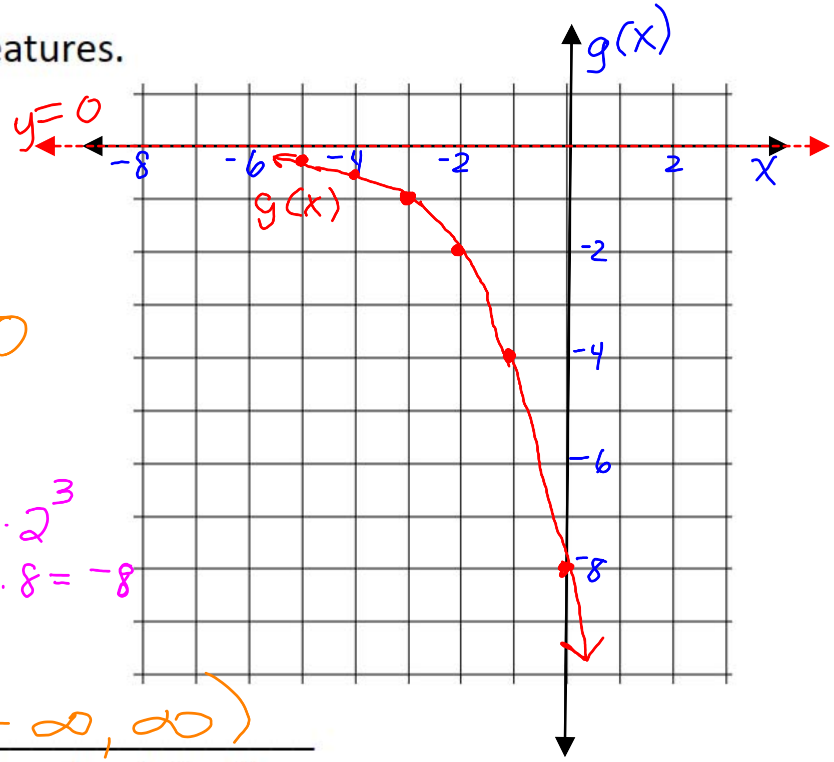
opp = h

$g(x) = -1 \cdot 2^{x+3}$

$a = -1$, $h = -3$, $k = 0$

x-axis refl. left

y-int → $g(0) = -1 \cdot 2^{0+3} = -1 \cdot 2^3 = -1 \cdot 8 = -8$



Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$

(set notation) (interval notation)

Range: $\{y \mid y < 0\}$ / $(-\infty, 0)$

(set notation) (interval notation)

y-intercept: $(0, -8)$

Horizontal Asymptote: $y = 0$

End Behavior: as $x \to -\infty$, $g(x) \to 0$

as $x \to \infty$, $g(x) \to -\infty$

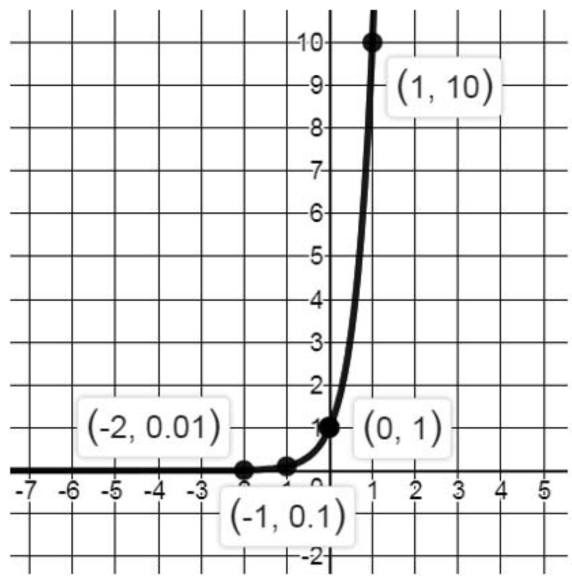
Objective: Graph and Identify Key Features of Exponential Growth Functions

Concept

The function $\boldsymbol{f(x) = 10^x}$ is the parent function for all exponential functions with base 10. These functions have the form $f(x) = a \cdot 10^{\frac{1}{b}(x-h)} + k$.

x	$f(x) = 10^x$
-2	$10^{-2} = \frac{1}{100} = 0.01$
-1	$10^{-1} = \frac{1}{10} = 0.1$
0	$10^0 = 1$
1	$10^1 = 10$
2	$10^2 = 100$

$\boldsymbol{f(x) = 10^x}$



Domain: the set of all real numbers; $\{x|-\infty < x < +\infty\}$; $(-\infty, +\infty)$
Range: $\{y|y > 0\}$; $(0, +\infty)$
Horizontal Asymptote: $y = 0$
End Behavior: $as\ x \to -\infty, f(x) \to 0; as\ x \to +\infty, f(x) \to +\infty$

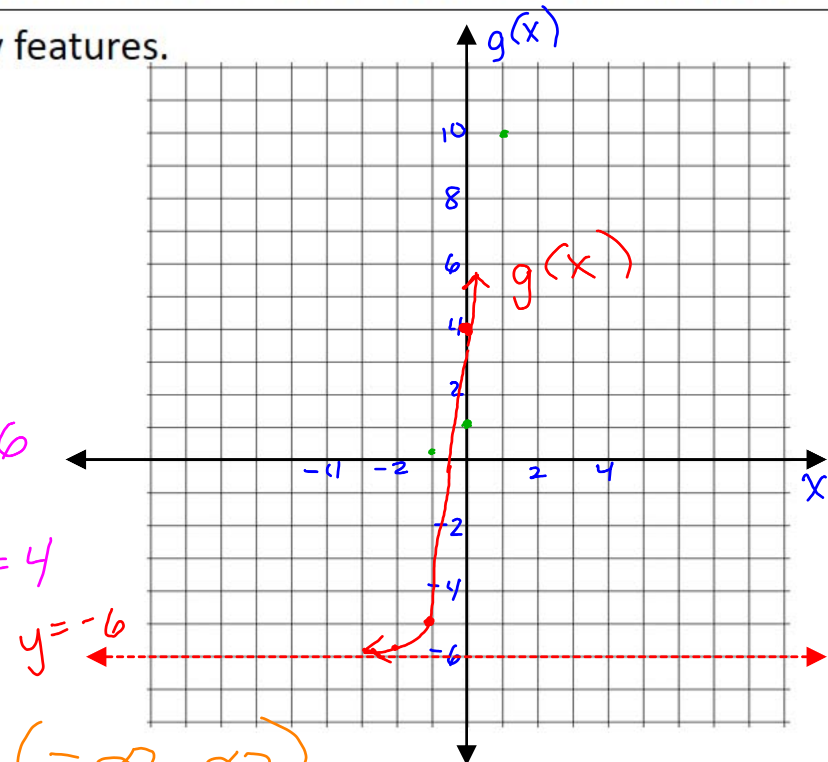
Objective: Graph and Identify Key Features of Exponential Growth Functions

Ex) Graph the function. State the key features.

$$g(x) = 10^{x+1} - 6$$

$a = 1$ $h = -1$ left $k = -6$ down

y-int → $g(0) = 10^{0+1} - 6$
$= 10^1 - 6$
$= 10 - 6 = 4$



Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y > -6\}$ / $(-6, \infty)$
(set notation) (interval notation)

y-intercept: $(0, 4)$

Horizontal Asymptote: $y = -6$

End Behavior: as $x \to -\infty$, $g(x) \to -6$
as $x \to \infty$, $g(x) \to \infty$

Objective: Graph and Identify Key Features of Exponential Growth Functions

Closure

Will an exponential function always have a y-intercept? Explain.

Yes, since the domain of an exponential function is the set of all real numbers, the x value of 0 will always produce a y-intercept.

Will an exponential function always have a zero? Explain.

No, since the horizontal asymptote can sometimes prevent the function from intersecting the x-axis, there will not always be a zero for an exponential function.