Objective: Know and Use the Properties of Logarithms

Concept

The **Properties of Logarithms** are valid for logarithms of all valid base values. This includes the common logarithm ($\log x$) and the natural logarithm ($\ln x$).

Properties of Logarithms	
For any positive numbers a, m, n, b $(b \neq 1)$, and c $(c \neq 1)$, the following properties hold.	
Power Property of Logarithms	$\log_b m^n = n \cdot \log_b m$
Product Property of Logarithms	$\log_b m + \log_b n = \log_b(m \cdot n)$
Quotient Property of Logarithm	$\log_b m - \log_b n = \log_b\left(\dfrac{m}{n}\right)$
Definition-Based Properties	$\log_b b = 1, \quad \log_b 1 = 0, \quad \log_b b^m = m$
Change of Base Property of Logarithms	$\log_c a = \dfrac{\log_b a}{\log_b c}$

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Ex) Use the properties of logarithms to simplify as much as possible. Show your work.

$$\frac{1}{2}\log 16 + 2\log 5$$

① power prop.

$$\log 16^{\frac{1}{2}} + \log 5^2$$

$$\log \sqrt{16} + \log 5^2$$

$$\log 4 \oplus \log 25$$

② product prop.

$$\log (4 \cdot 25)$$

$$\log 100$$

③ $\log_b b^m = m$

$$\log_{10} 10^2$$

$$= \boxed{2}$$

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Ex) Use the properties of logarithms to simplify as much as possible. Show your work.

$$log_3 27 \ominus log_3 81$$

① quotient prop.

$$log_3 \left(\frac{27}{81} \right)$$

$$log_3 \left(\frac{1}{3^1} \right)$$

② $log_b b^m = m$

$$log_3 3^{-1}$$

$$= \boxed{-1}$$

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Ex) Use the properties of logarithms to simplify as much as possible. Show your work.

$$\ln 10 + 3 \ln 2$$

① power prop.

$$\ln 10 + \ln 2^3$$

$$\ln 10 + \ln 8$$

② product prop.

$$\ln (10 \cdot 8)$$

$$\boxed{\ln 80}$$

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Ex) Use the properties of logarithms to simplify as much as possible. Show your work.

$$4 \log_4 2 + \frac{1}{3} \log_4 27 - \log_4 6 = \frac{3}{2}$$

① power prop.

$$\log_4 2^4 + \log_4 27^{\frac{1}{3}} - \log_4 6$$
$$\sqrt[3]{27}$$

$$\log_4 16 \oplus \log_4 3 \ominus \log_4 6$$

② product prop.

$$\log_4 (16 \cdot 3)$$

$$\log_4 48 \ominus \log_4 6$$

③ quotient prop.

$$\log_4 \left(\frac{48}{6}\right)$$

$$\log_4 8 = n$$

④ exponential form

$$4^n = 8$$

$$(2^2)^n = 2^3$$

$$2^{2n} = 2^3$$

$$2n = 3$$

$$n = \frac{3}{2}$$

Objective: Know and Use the Properties of Logarithms

Practice) Use the properties of logarithms to simplify as much as possible. Show your work.

$$3\ln 2 - \frac{4}{3}\ln 8 + \ln 2$$

$$= \ln 2^3 - \ln 8^{\frac{4}{3}} + \ln 2 \qquad \text{power property of logarithms}$$

$$= \ln 8 - \ln 16 + \ln 2 \qquad \text{simplify}$$

$$= \ln\left(\frac{8}{16}\right) + \ln 2 \qquad \text{quotient property of logarithms}$$

$$= \ln\left(\frac{1}{2}\right) + \ln 2 \qquad \text{simplify}$$

$$= \ln\left(\frac{1}{2} \cdot 2\right) \qquad \text{product property of logarithms}$$

$$= \ln(1) \qquad \text{simplify}$$

$$= \boxed{0} \qquad \text{definition-based property: } \log_b 1 = 0$$