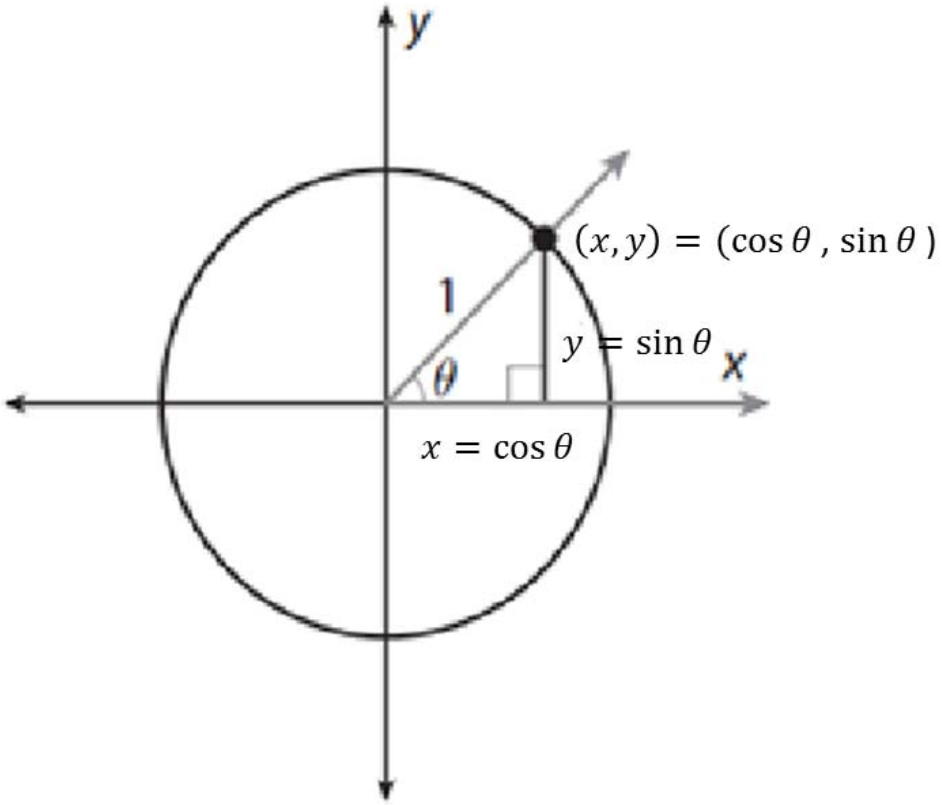
Objective: Simplify trigonometric expressions.

Concept

The **Pythagorean Identities** are based on the Pythagorean Theorem and its application to the Unit Circle.



$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = 1^2$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

or

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Objective: Simplify trigonometric expressions.

Concept

From the first Pythagorean Identity other identities can be derived.

dividing by $\sin^2\theta$:

$$\sin^2\theta+\cos^2\theta=1$$

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$1+\cot^2\theta=\csc^2\theta$$

dividing by $\cos^2\theta$:

$$\sin^2\theta+\cos^2\theta=1$$

$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$\tan^2\theta+1=\sec^2\theta$$

Objective: Simplify trigonometric expressions.

Fundamental Trigonometric Identities

Pythagorean Identities

★memOrize

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Objective: Simplify trigonometric expressions.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Objective: Simplify trigonometric expressions.

Concept

Strategies for Simplifying a Trigonometric Expression

1. Substitute using identities.
2. Factor.
3. Distribute.
4. Get a common denominator.
5. Multiply by a ratio of 1.
6. Rewrite in terms of sine and cosine.

Recognizing a Simplified Trigonometric Expression

1. The expression is written in terms of a single trigonometric function.
2. The expression does not involve any fraction forms.
3. No more identities can be used to further simplify the expression.

Objective: Simplify trigonometric expressions.

Simplify the expression.

$\sin x \cos^2 x - \sin x$

$\sin x \cdot \cos^2 x - 1\sin x$

① factor

$\sin x(\cos^2 x - 1)$

$\sin x \cdot -1(-\cos^2 x + 1)$

② identity

$\sin^2\theta = 1 - \cos^2\theta$

$-\sin x(1 - \cos^2 x)$

$-\sin x \cdot \sin^2 x$

$-\sin^3 x$

Objective: Simplify trigonometric expressions.

Simplify the expression.

$$\cos^2 x \csc x - \csc x$$

① factor

$\csc x(\cos^2 x - 1)$

$\csc x \cdot (-\sin^2 x)$

$\frac{1}{\sin x} \cdot \frac{-\sin^2 x}{1}$

$\frac{1}{\sin x} \cdot \frac{-1 \cdot \sin x \cdot \sin x}{1}$

$-\sin x$

② identity

$-1 \cdot (\sin^2 x = 1 - \cos^2 x)$

$-\sin^2 x = -1 + \cos^2 x$

$-\sin^2 x = \cos^2 x - 1$

Objective: Simplify trigonometric expressions.

Simplify the expression.

$$\sin x + \cot x \cos x$$

① $\sin x + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{1}$

$\sin x + \frac{\cos^2 x}{\sin x}$

② common denominator $\frac{\sin x}{1} \cdot \frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$

③ add $\frac{\sin^2 x + \cos^2 x}{\sin x}$

④ identity

⑤ identity $\frac{1}{\sin x}$

$\csc x$

Objective: Simplify trigonometric expressions.

Simplify the expression.

$$\csc x - \cos x \cot x$$

$$\csc x - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$identity: \cot x = \frac{\cos x}{\sin x}$$

$$\csc x - \frac{\cos^2 x}{\sin x}$$

$$identity: \csc x = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$identity: 1 - \cos^2 x = \sin^2 x$$

$$\frac{\sin^2 x}{\sin x} \rightarrow \frac{(\sin x)\cancel{^1}}{\cancel{\sin x}} \rightarrow \boxed{\sin x}$$

Objective: Simplify trigonometric expressions.

Ex) Factor the expression.

$$\sec^2\theta - 1$$

① difference of two squares

structure $x^2 - 1$

$(x)^2 - (1)^2$

$(x+1)(x-1)$

$(\sec\theta)^2 - (1)^2$

$(\sec\theta - 1)(\sec\theta + 1)$

check: $\sec^2\theta + \sec\theta - \sec\theta - 1$

Objective: Simplify trigonometric expressions.

Ex) Factor the expression.

$$4\tan^2\theta + \tan\theta - 3$$

$4\tan\theta, \tan\theta$
$2\tan\theta, 2\tan\theta$

★ quadratic trinomial structure

$$(4\tan\theta - 3)(\tan\theta + 1)$$

$4x^2 + x - 3$
$(4x-3)(x+1)$

check: $4\tan^2\theta + 4\tan\theta - 3\tan\theta - 3$
$+ \tan\theta$ ✓

Objective: Simplify trigonometric expressions.

Practice) Factor the expression.

$$1 - \cos^2\theta$$

structure

$1 - x^2$

$(1+x)(1-x)$

$(1)^2 - (\cos\theta)^2$

$(1 + \cos\theta)(1 - \cos\theta)$

check: $1 - \cos\theta + \cos\theta - \cos^2\theta$

Objective: Simplify trigonometric expressions.

Practice) Factor the expression.

$2\csc^2\theta - 7\csc\theta + 6$

$-1, -6$
$-2, -3$

structure
$2x^2 - 7x + 6$
$(2x-3)(x-2)$

$(2\csc\theta - 3)(\csc\theta - 2)$

$2\csc^2\theta - 4\csc\theta - 3\csc\theta + 6$
$-7\csc\theta$

Objective: Simplify trigonometric expressions.

Practice) Factor the expression.

structure

$6x^2 - 3x$

$3x(2x - 1)$

$6\sin^2\theta - 3\sin\theta$

$3\sin\theta(2\sin\theta - 1)$

check: $6\sin^2\theta - 3\sin\theta$

Objective: Simplify trigonometric expressions.

Ex) Factor the expression.

$$\csc^2 x - \cot x - 3$$

① substitution for $\csc^2 x$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

② structure

$$x^2 - x - 2$$

$$(x-2)(x+1)$$

$$1 + \cot^2 x - \cot x - 3$$

$$\cot^2 x - \cot x - 2$$

$$(\cot x - 2)(\cot x + 1)$$

check. $\cot^2 x + \cot x - 2\cot x - 2$

$-\cot x$

Objective: Simplify trigonometric expressions.

Practice) Factor the expression.

$$\sec^2 x + 3\tan x + 1$$

① $\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

② structure

$x^2 + 3x + 2$

$(x+2)(x+1)$

$\tan^2 x + 1 + 3\tan x + 1$

$\tan^2 x + 3\tan x + 2$

$(\tan x + 2)(\tan x + 1)$

Objective: Simplify trigonometric expressions.

Ex) Perform the addition and simplify.

$$\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

① get a common den.

$$\frac{\sin\theta}{(1+\cos\theta)}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}+\frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{\sin\theta}$$

②

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta$

$$\frac{\sin\theta-\sin\theta\cdot\cos\theta}{1-\cos^2\theta}+\frac{\cos\theta\cdot\sin\theta}{\sin^2\theta}$$

$$\frac{\sin\theta-\sin\theta\cos\theta}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$$

$$\frac{\sin\theta-\cancel{\sin\theta\cos\theta}+\cancel{\sin\theta\cos\theta}}{\sin^2\theta}\quad(0)$$

③ reduce structure

$$\frac{x}{x^2}=\frac{1}{x}$$

$x\cdot x$

$$\frac{\sin\theta}{\sin^2\theta}\rightarrow\frac{1}{\sin\theta}\rightarrow\boxed{\csc\theta}$$

Objective: Simplify trigonometric expressions.

Practice) Perform the addition and simplify.

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$\frac{1}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}+\frac{1}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$\frac{1-\sin\theta}{1-\sin^2\theta}+\frac{1+\sin\theta}{1-\sin^2\theta}$$

$$\frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$$

$$identity: \cos^2\theta = 1-\sin^2\theta$$

$$\frac{1-\cancel{\sin\theta}+1+\cancel{c\sin\theta}}{\cos^2\theta}$$

$$identity: \frac{1}{\cos\theta}=\sec\theta$$

$$\frac{2}{\cos^2\theta}\rightarrow\frac{2}{1}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$2\cdot\sec\theta\cdot\sec\theta\rightarrow\boxed{2\sec^2\theta}$$