Objective: Find the average rate of change of quadratic functions given context.

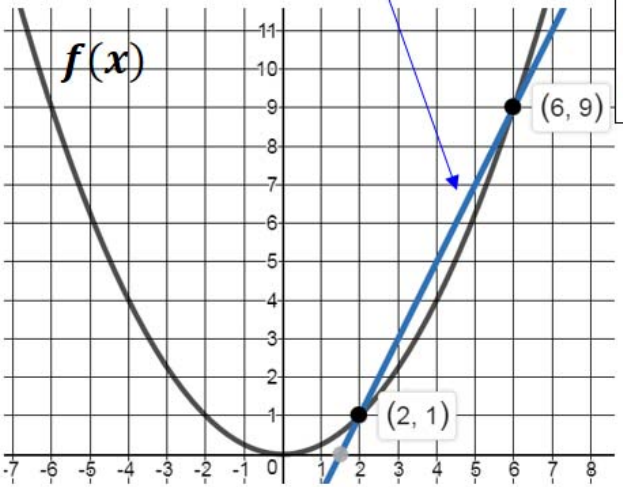
Concept

Average Rate of Change for a Function $f(x)$

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x} = \frac{change\ (difference)\ in\ the\ y\ values}{change\ (difference)\ in\ the\ x\ values}$$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ for the interval } [x_1, x_2]$$

AROC for the interval $[2,6]$ is the slope of this line

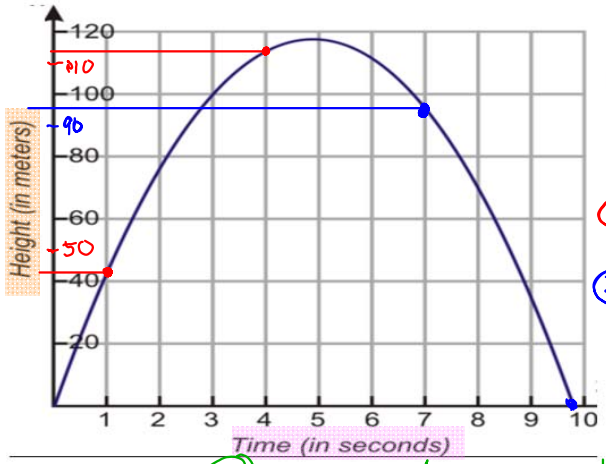


The **average rate of change** of a function for an interval **is equivalent to the slope of the line through the two points of the function**. This line is called the secant line.

For the function $f(x)$ on the interval $[2,6]$ the average rate of change is $\frac{\Delta y}{\Delta x} = \frac{9-1}{6-2} = \frac{8}{4} = 2$

Objective: Find the average rate of change of quadratic functions given context.

Ex) The graph shows the height of a rocket, $h(t)$, in meters, t seconds after it is launched.



a) Approximate the average speed of the rocket from 1 second to 4 seconds and interpret the meaning in context. Round to three decimal places.

① points: (1 sec, 42 meters), (4 sec, 114 meters)

② AROC

$$\frac{\Delta h(t)}{\Delta t} = \frac{114 \text{ meters} - 42 \text{ meters}}{4 \text{ sec} - 1 \text{ sec}} = \frac{72 \text{ meters}}{3 \text{ sec}}$$

m/sec

$\approx$ 24 meters per second

* positive means the rocket is rising

③ From 1 second to 4 seconds the rocket is rising at an average speed of about 24 meters per second.

b) Approximate the rocket's average speed from 7 seconds until the rocket hits the ground and interpret the meaning in context. Round to three decimal places.

① points: (7 sec, 96 m) (9.8 sec, 0 m)

② AROC

$$\frac{\Delta h(t)}{\Delta t} = \frac{0 \text{ m} - 96 \text{ m}}{9.8 \text{ sec} - 7 \text{ sec}} = \frac{-96 \text{ meters}}{2.8 \text{ sec}}$$

≈ -34.286 meters per second

* speed is always positive

③ From 7 seconds until the rocket hits the ground the rocket is falling at an average speed of about 34.286 meters per second.

* negative means the rocket is falling

Objective: Find the average rate of change of quadratic functions given context.

Ex) The path of a baseball thrown into the air can be modeled by the function $p(x) = -0.01x^2 + 0.5x + 3.75$, where x is the horizontal distance traveled, in feet, from the release point, and y is the corresponding height of the baseball in feet.

A) Find the average change in the baseball's height per foot of horizontal distance traveled from 0 feet to 12 feet of distance traveled. Round to three decimal places if necessary. Interpret in terms of the context.

① points: (0 ft distance, 3.75 ft height) (12 ft distance, 8.31 ft height)

$p(0) = -0.01(0)^2 + 0.5(0) + 3.75$ $p(12) = -0.01(12)^2 + 0.5(12) + 3.75$

② AROC

$$\frac{\Delta p(x)}{\Delta x} = \frac{8.31 \text{ ft} - 3.75 \text{ ft}}{12 \text{ ft} - 0 \text{ ft}} = \frac{4.56 \text{ ft (height)}}{12 \text{ ft distance}}$$

$= 0.38$ feet per foot of distance

* positive means the baseball is rising.

③ From 0 feet to 12 feet of distance traveled the baseball is rising at an average of 0.38 feet per foot of distance.

Objective: Find the average rate of change of quadratic functions given context.

Ex) The table represents the velocity, V, of an object, in meters per second, at a time t, in seconds.

t Sec	$V(t)$ m/sec
0	60
★ 1	35
2	20
★ 3	15
4	20
5	35
6	60
7	95

A) What is the average acceleration (meters per second per second) of the object from 1 second to 3 seconds? Round to three decimal places if necessary. Interpret in terms of the context.

① points: (1 sec, 35 m/s), (3 sec, 15 m/s)

② AROC:

$$\frac{\Delta V(t)}{\Delta t} = \frac{15\text{m/s} - 35\text{m/s}}{3\text{sec} - 1\text{sec}} = \frac{-20\text{m/s}}{2\text{sec}}$$

$= -10$ m/sec/sec
meters per second per second

★ negative means the object is slowing down. or decelerating

★ acceleration is always a positive value/quantity.

③ From 1 second to 3 seconds the object is decelerating at an average of 10 meters per second per second.

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Closure

A toy rocket is thrown straight up. Its height after 1 second is 10 feet. If the rocket's average speed from 1 second to 3 seconds is 4 feet per second. What is the height of the rocket after 3 seconds?

Time	Height
$1\ sec$	$10\ ft$
$2\ sec$	$10 + 4 = 14\ ft$
$3\ sec$	$14 + 4 = 18$ ft

The height of the rocket after 3 seconds is 18 feet.