Objective: Create a polynomial function in factored form from a graph.

Concept

Many functions can be written in multiple forms. Two **common forms of polynomial functions are standard form and factored form, also referred to as intercept form**.

Standard Form		Factored Form/Intercept Form
$f(x) = x^2 + 3x - 18$	$\longleftrightarrow$	$f(x) = (x+6)(x-3)$
$g(x) = -2x^2 + 8x + 10$	$\longleftrightarrow$	$g(x) = -2(x-5)(x+1)$

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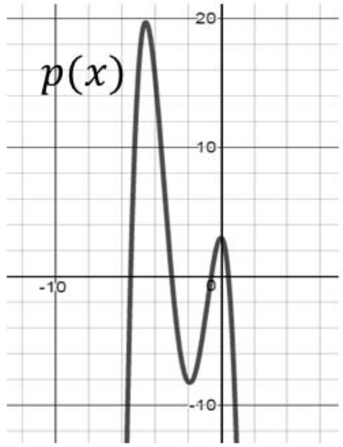
Concept

For polynomial functions, the end behavior can be used to determine the sign of the leading coefficient, or constant factor a.

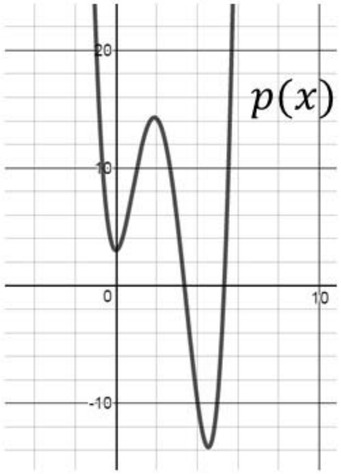


End Behavior

$as\ x \to -\infty, p(x) \to -\infty$

$as\ x \to +\infty, p(x) \to -\infty$

→ Leading coefficient (constant factor a) is negative.



End Behavior

$as\ x \to -\infty, p(x) \to +\infty$

$as\ x \to +\infty, p(x) \to +\infty$

→ Leading coefficient (constant factor a) is positive.

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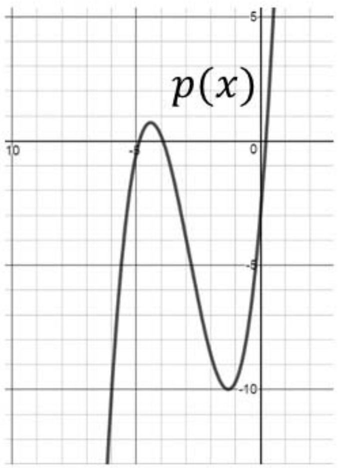
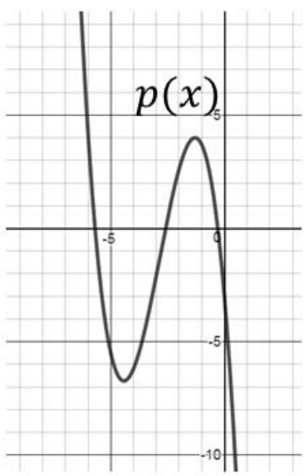
For polynomial functions, the end behavior can be used to determine the sign of the leading coefficient, or constant factor a.



End Behavior
$as\ x \to -\infty, p(x) \to -\infty$
$as\ x \to +\infty, p(x) \to +\infty$
→
Leading coefficient (constant factor a) is positive.



End Behavior
$as\ x \to -\infty, p(x) \to +\infty$
$as\ x \to +\infty, p(x) \to -\infty$
→
Leading coefficient (constant factor a) is negative.

Objective: Create a polynomial function in factored form from a graph.

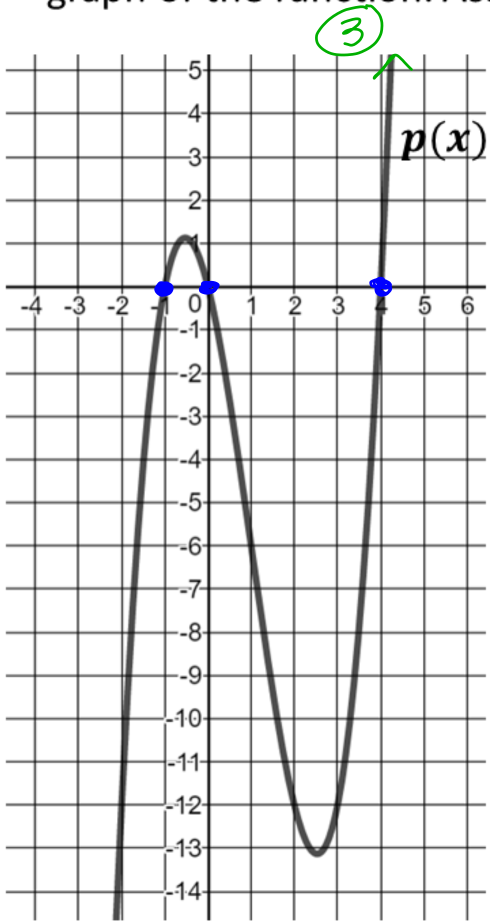
Concept

Steps to create a polynomial function of least degree in factored form from a graph.

1. Determine the zeros of the function. Include multiplicity.
2. Find the factors of the function using the zeros. Include factors that repeat.
3. Determine the sign of the leading coefficient (positive or negative) using the end behavior.
4. Write the polynomial function.

Objective: Create a polynomial function in factored form from a graph.

Ex) Write a polynomial function $p(x)$ of least degree in factored form using the graph of the function. Assume that the constant factor a is 1 or -1.



① zeros
$x = -1, 0, 4$ (no multiplicity)

② factors

$x = -1$
$+1 \quad +1$
$x + 1 = 0$
factor

$x = 0$
factor

$x = 4$
$-4 \quad -4$
$x - 4 = 0$
factor

③ sign of leading coefficient

as $x \to +\infty$, $p(x) \to (+)\infty$
$a = 1$

④ write the function

$p(x) = 1x(x+1)(x-4)$

$p(x) = x(x+1)(x-4)$

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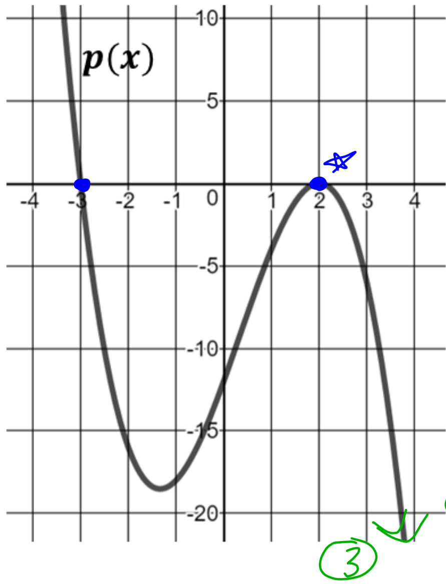
Ex) Write a polynomial function $p(x)$ of least degree in factored form using the graph of the function. Assume that the constant factor a is 1 or -1.



① zeros
-3, 2 (multiplicity × 2)
$x = -3, 2, 2$

② factors

$x = -3$ → $+3$ $+3$ → $x + 3 = 0$ factor

$x = 2$ → -2 -2 → $x - 2 = 0$ factor

$x = 2$ → -2 -2 → $x - 2 = 0$ factor

③ sign of leading coefficient
as $x \to +\infty$, $p(x) \to (-)\infty$
$a = -1$

④ write the function
$p(x) = -1(x+3)(x-2)(x-2)$

$$p(x) = -(x+3)(x-2)^2$$

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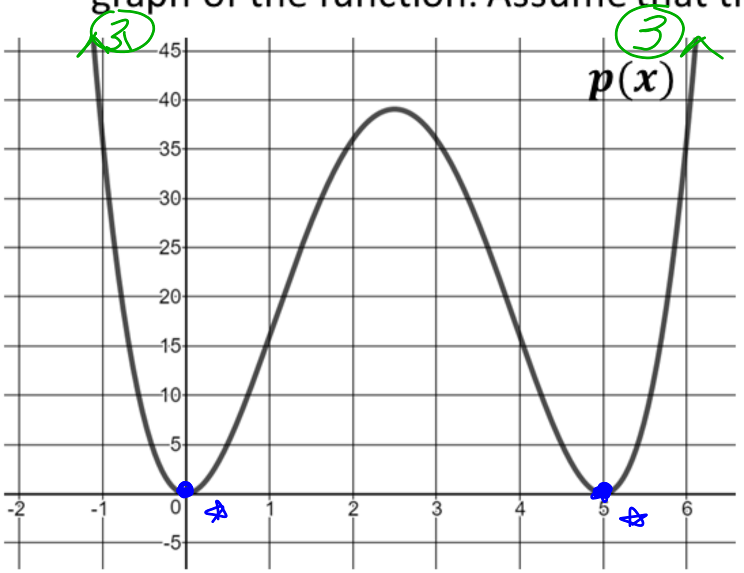
Ex) Write a polynomial function $p(x)$ of least degree in factored form using the graph of the function. Assume that the constant factor a is 1 or -1.



① zeros

0 (mult. x2), 5 (mult. x2)

$x = 0, 0, 5, 5$

② factors

$x = 0$, $x = 0$, $x = 5$, $x = 5$

factor, factor

$-5\ -5$, $-5\ -5$

$x - 5 = 0$, $x - 5 = 0$

factor, factor

③ sign of the leading coefficient

$p(x) \to (+)\infty$ on both ends

$a = 1$

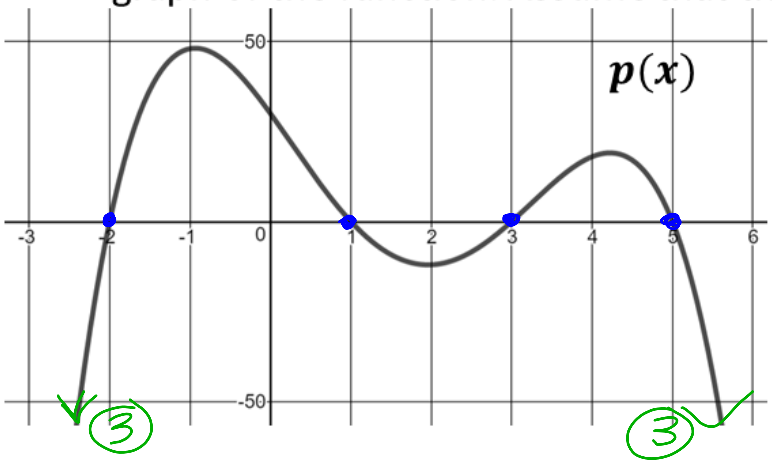
④ write the function

$p(x) = 1x \cdot x \cdot (x-5)(x-5)$

$$p(x) = x^2(x-5)^2$$

Objective: Create a polynomial function in factored form from a graph.

Ex) Write a polynomial function $p(x)$ of least degree in factored form using the graph of the function. Assume that the constant factor a is 1 or -1.



① zeros

$x = -2,\ 1,\ 3,\ 5$ (no mult.)

② factors

$x = -2$, $+2\ \ +2$, $x + 2 = 0$

$x = 1$, $-1\ \ -1$, $x - 1 = 0$

$x = 3$, $-3\ \ -3$, $x - 3 = 0$

$x = 5$, $-5\ \ -5$, $x - 5 = 0$

③ sign of the leading coefficient

$p(x) \to (-)\infty$ on both ends

$a = -1$

④ write the function

$p(x) = -1(x+2)(x-1)(x-3)(x-5)$

$p(x) = -(x+2)(x-1)(x-3)(x-5)$

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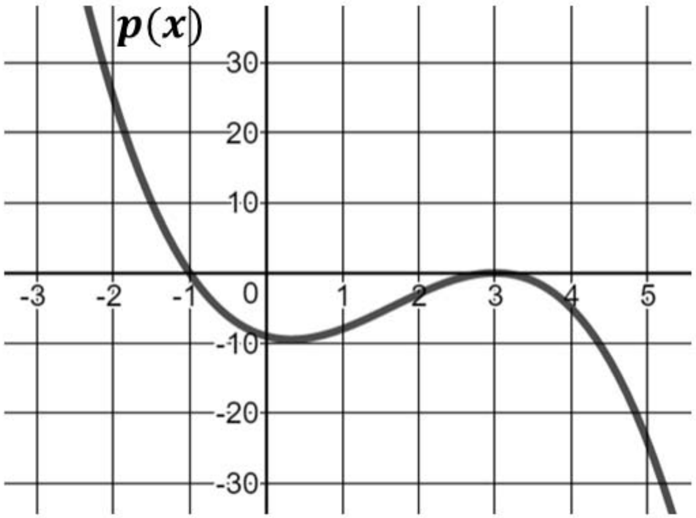
Closure

Bethany created a polynomial function for the function shown in the graph. Bethany made two errors. Determine her errors and explain how to fix them to create the correct function. Then, write the correct function.



Bethany's Function:

$$p(x) = -(x-1)^2(x-3)$$

One of Bethany's errors is that the factor of $(x-1)$ should be $(x+1)$. Her second error is that the factor of $(x-3)$ should be the squared factor because the zero of 3 has a multiplicity of 2. The correct function is $p(x) = -1(x+1)(x-3)^2$.