Objective: Add and Subtract Rational Expressions.

Concept

A rational expression has the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials. Since there will be a variable in the denominator, a rational expression has excluded values. These are values of x that would make the denominator equal to 0. Denominators can't be equal to 0 because that represents division by 0 which is undefined (not mathematically possible).

Excluded Values of a Rational Expression

To find the excluded values of a rational expression, find the values of the variable that make the denominator equal to 0.

For $\frac{x+3}{x^2-5x-6}$, solve for where $x^2-5x-6=0$

$$(x-6)(x+1)=0$$

$$x-6=0,\ x+1=0$$

$$x=6 \qquad x=-1$$

Conclusion: $x \neq 6,\ x \neq -1$

Objective: Add and Subtract Rational Expressions.

Concept

We add and subtract rational expressions using the same basic procedure used to add and subtract numeric fractions.

To Add and Subtract Rational Expressions:

1. Find the lowest common denominator (LCD).

2. Multiply each rational expression by a ratio of 1 using missing factors in the LCD.

3. Add or subtract the numerators of the rational expressions; place this over the denominator of the LCD.

Objective: Add and Subtract Rational Expressions.

Ex) Add or subtract the given expressions. State any excluded values.

$$\frac{2x}{x^2+9x+20} - \frac{5}{3x+12}$$

$$\frac{2x}{(x+5)(x+4)} \quad \frac{5}{3(x+4)}$$

(a) excluded values

$x+5=0$, $x+4=0$, $3 \neq 0$

$x \neq -5$ $x \neq -4$

or

$x \neq -5, -4$

(b)

$$\frac{2x}{(x+5)(x+4)} - \frac{5}{3(x+4)}$$

① LCD $= 3(x+4)(x+5)$

② $\frac{2x}{(x+5)(x+4)} \cdot \frac{3}{3} - \frac{5}{3(x+4)} \cdot \frac{(x+5)}{(x+5)}$

③ $\frac{6x}{3(x+5)(x+4)} - \frac{(5x+25)}{3(x+5)(x+4)}$

④ $\frac{6x - 5x - 25}{3(x+5)(x+4)}$

⑤ reduce if possible (can't here)

$$\frac{x-25}{3(x+5)(x+4)}$$

Objective: Add and Subtract Rational Expressions.

Ex) Add or subtract the given expressions. State any excluded values.

$$\frac{x+4}{4x^2-9} - \frac{x-2}{2x^2-3x}$$

$(2x+3)(2x-3)$ $\quad x(2x-3)$

@excluded values

$2x+3=0$, $2x-3=0$, $x=0$

$x \neq \frac{-3}{2}$ $\quad x \neq \frac{3}{2}$ $\quad x \neq 0$

$$x \neq \frac{-3}{2}, 0, \frac{3}{2}$$

(b) $\dfrac{x+4}{(2x+3)(2x-3)} - \dfrac{x-2}{x(2x-3)}$

① LCD = $x(2x-3)(2x+3)$

② $\dfrac{(x+4)}{(2x+3)(2x-3)} \cdot \dfrac{x}{x} - \dfrac{(x-2)}{x(2x-3)} \cdot \dfrac{(2x+3)}{(2x+3)}$

③ $\dfrac{x^2+4x}{x(2x+3)(2x-3)} - \dfrac{(2x^2-x-6)}{x(2x+3)(2x-3)}$

④ $\dfrac{x^2+4x-2x^2+(+x)+(+6)}{x(2x+3)(2x-3)}$

⑤ reduce if possible (can't reduce)

$$\frac{-x^2+5x+6}{x(2x+3)(2x-3)}$$

Objective: Add and Subtract Rational Expressions.

Ex) Add or subtract the given expressions. State any excluded values.

$$\frac{2x-4}{x^2-5x} + \frac{x+6}{x^2}$$

$x(x-5)$ $x \cdot x$

(a) excluded values

$x = 0$, $x - 5 = 0$

$x \neq 0$ $x \neq 5$

$x \neq 0, 5$

(b) $\frac{2x-4}{x(x-5)} + \frac{x+6}{x \cdot x}$

(1) LCD $= x \cdot x (x-5)$
$= x^2(x-5)$

(2) $\frac{(2x-4)}{x(x-5)} \cdot \frac{x}{x} + \frac{(x+6)}{x \cdot x} \cdot \frac{(x-5)}{(x-5)}$

(3) $\frac{2x^2-4x}{x^2(x-5)} + \frac{(x^2+x-30)}{x^2(x-5)}$

(4) $\frac{2x^2-4x+x^2+x+-30}{x^2(x-5)}$

(5) reduce if possible (can't here)

$\frac{3x^2-3x-30}{x^2(x-5)}$

Objective: Add and Subtract Rational Expressions.

Ex) Add or subtract the given expressions. State any excluded values.

$$\frac{-4}{x-4}+\frac{3x}{x+3}$$

(a) excluded values

$x-4=0$ $\quad x+3=0$

$x\neq 4$ $\quad x\neq -3$

$x\neq -3,4$

(b)

(1) LCD $=(x-4)(x+3)$

$$\frac{-4}{(x-4)}+\frac{3x}{(x+3)}$$

(2) $$\frac{-4}{(x-4)}\cdot\frac{(x+3)}{(x+3)}+\frac{3x}{(x+3)}\cdot\frac{(x-4)}{(x-4)}$$

(3) $$\frac{-4x-12}{(x-4)(x+3)}+\frac{(3x^2-12x)}{(x+3)(x-4)}$$

(4) $$\frac{-4x-12+3x^2+-12x}{(x-4)(x+3)}$$

(5) reduce if possible (can't here)

$$\frac{3x^2-16x-12}{(x-4)(x+3)}$$

Objective: Add and Subtract Rational Expressions.

Ex) Add or subtract the given expressions. State any excluded values.

$$\frac{-14}{x^2-11x+24}+\frac{x+2}{x^2-6x+9}$$

$$\frac{-14}{(x-8)(x-3)}+\frac{x+2}{(x-3)(x-3)}$$

(a) excluded values

$x-8=0,\ x-3=0$

$x \neq 3, 8$

(b)

$$\frac{-14}{(x-8)(x-3)}+\frac{x+2}{(x-3)(x-3)}$$

① LCD = $(x-3)(x-3)(x-8)$

②

$$\frac{-14}{(x-8)(x-3)}\cdot\frac{(x-3)}{(x-3)}+\frac{(x+2)}{(x-3)(x-3)}\cdot\frac{(x-8)}{(x-8)}$$

③

$$\frac{-14x+42}{(x-8)(x-3)^2}+\frac{(x^2-6x-16)}{(x-8)(x-3)^2}$$

④

$$\frac{-14x+42+x^2+-6x+-16}{(x-3)^2(x-8)}$$

⑤ reduce if possible (can't reduce here)

$$\frac{x^2-20x+26}{(x-3)^2(x-8)}$$

Objective: Add and Subtract Rational Expressions.

Closure

A set of numbers or expressions is said to be closed, or to have **closure**, under a given operation if the result of the operation on any two numbers or expressions in the set is also in the set.

The **Set of Rational Expressions** includes all **expressions of the form** $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

Is the Set of Rational Expressions closed under addition?

Yes, because the sum of two rational expressions will always be another rational expression.

$$\frac{p(x)}{q(x)} + \frac{r(x)}{s(x)} = \frac{p(x)s(x) + q(x)r(x)}{q(x)s(x)}$$

Is the Set of Rational Expressions closed under subtraction?

Yes, because the difference of two rational expressions will always be another rational expression.

$$\frac{p(x)}{q(x)} - \frac{r(x)}{s(x)} = \frac{p(x)s(x) - q(x)r(x)}{q(x)s(x)}$$