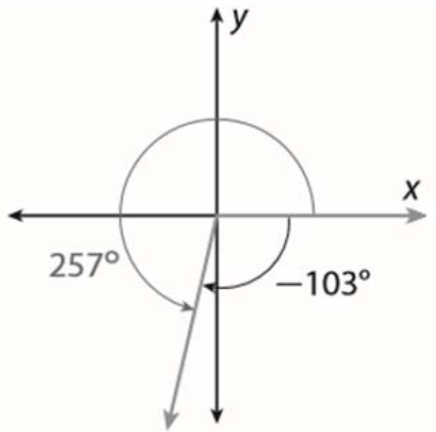
Objective: Find Coterminal Angles

Concept

Coterminal angles are angles that share the same terminal side.

For example: the angles with measures $257°$ and $-103°$ are coterminal, as shown.



Since any number of revolutions for an angle of rotation are allowed, given an angle θ, there are an infinite number of angles coterminal with θ. **The angles that are coterminal with θ are written as:**

$$\theta + 360k^\circ \text{ or } \theta + 2\pi k, \text{ where } k \text{ is any integer.}$$

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Ex) Write an expression that represents all angles coterminal with the given angle.

$210°$

$= \theta$

$210° + 360k°$ where k is any integer

$\frac{5\pi}{4} = \theta$

$\frac{5\pi}{4} + 2\pi k$ where k is any integer

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Ex) Find the angles coterminal with the given angle for k values of $-1, 1,$ and 2.

$225°$

① $225° + 360k°$

② $k = -1$

$225° + -360°$

$360 - 225$

$= -135°$

③ $k = 1$

$225° + 360°$

$= 585°$

④ $k = 2$

$225° + 720°$

$= 945°$

$\frac{\pi}{3}$

① $\frac{\pi}{3} + 2\pi k$

② $k = -1$

$\frac{\pi}{3} + -2\pi$

$\frac{1}{3}\pi + -2\pi$

$\frac{1}{3}\pi + \frac{-6}{3}\pi$

$= -\frac{5}{3}\pi = -\frac{5\pi}{3}$

③ $k = 1$

$\frac{\pi}{3} + 2\pi$

$\frac{1}{3}\pi + 2\pi = 2\frac{1}{3}\pi$

$= \frac{7}{3}\pi = \frac{7\pi}{3}$

④ $k = 2$

$\frac{\pi}{3} + 4\pi$

$\frac{1}{3}\pi + 4\pi$

$= 4\frac{1}{3}\pi = \frac{13}{3}\pi$

$= \frac{13\pi}{3}$

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Ex) Find the angle in the first revolution that is coterminal with the given angle.

$0 \le \theta \le 2\pi$

$\frac{5\pi}{2}$

① $= \frac{5}{2}\pi$

$= 2\frac{1}{2}\pi$

② $\frac{-2\pi}{\frac{1}{2}\pi}$

③ $\frac{\pi}{2}$

$-\frac{5\pi}{3}$

① $-\frac{5}{3}\pi$

$= -1\frac{2}{3}\pi$

② $\frac{+2\pi}{\frac{1}{3}\pi}$

$\left(-\frac{5}{3}\pi + \frac{6}{3}\pi\right)$

③ $\frac{\pi}{3}$

$\frac{21\pi}{4}$

① $5\frac{1}{4}\pi$

② $\frac{-4\pi}{1\frac{1}{4}\pi}$

③ $\frac{5\pi}{4}$

Objective: Find Coterminal Angles

Ex) Find the angle in the first revolution that is coterminal with the given angle.

$0° \leq \theta < 360°$

$-580°$

$+360°$

$-220°$

$+360°$

$140°$

$600°$

$-360°$

$240°$

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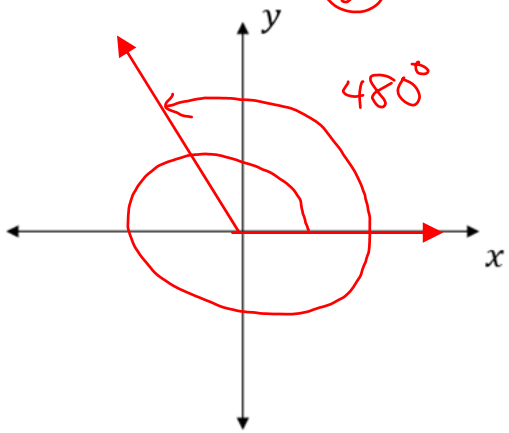
Ex) Draw the angle in standard position.

$480°$

① coterminal angle in the first revolution

$$\begin{array}{r} 480^\circ \\ -\ 360^\circ \\ \hline 120^\circ \end{array}$$

②



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Ex) Draw the angle in standard position.

$$-\frac{29\pi}{6}$$

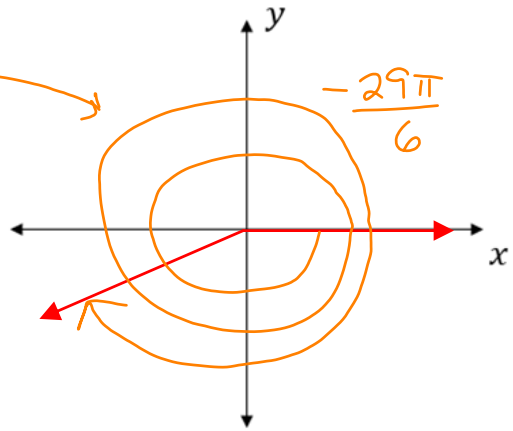
① coterminal angle in the first revolution

$$-\frac{29\pi}{6} = -4\frac{5}{6}\pi$$ ★

$$+6\pi$$

$$1\frac{1}{6}\pi = \frac{7}{6}\pi$$

or $$-\frac{29\pi}{6} + \frac{36\pi}{6}$$



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Closure

Sharon was asked to find the angle in the first revolution that is coterminal with $-400°$. Her work is shown. Do you agree or disagree with Sharon's answer? Explain your reasoning.

$$-400^\circ + 360^\circ = -40^\circ$$

The angle in the first revolution coterminal with $-400°$ is $40°$.

I disagree with Sharon's answer. She should have added another $360°$ to the $-40°$ to get an angle of $320°$ for the coterminal angle.