Objective: Find key characteristics of rational functions.

Concept

To Find All Vertical Asymptotes and Holes

1. From the form $f(x) = \frac{p(x)}{q(x)}$. Factor both numerator and denominator of the function and reduce, if possible.

2. **A common factor** between numerator and denominator **indicates there will be a hole (a point of discontinuity)** with an x-value that makes the factor equal to 0. The y-value of the hole is found by evaluating the simplified function using the x-value.

3. **Factors in the denominator that do not have a matching factor** in the numerator **indicate there will be a vertical asymptote at $x = c$**, where c is the value of x that makes the factor equal to 0.

Objective: Find key characteristics of rational functions.

Concept

To Find the Horizontal Asymptote (if one exists):	
Use the simplified form of the function and the information below.	
degree of $p(x) <$ degree of $q(x)$	horizontal asymptote: $y = 0$
degree of $p(x) =$ degree of $q(x)$	horizontal asymptote: $y = \frac{a}{b}$, where a and b are the leading coefficients of the numerator and denominator
degree of $p(x) >$ degree of $q(x)$	No horizontal asymptote exists; instead, there is a slant (oblique) asymptote

Remember: The degree of a polynomial expression is the greatest exponent value on the variable.

Objective: Find key characteristics of rational functions.

Concept

Finding x-intercepts/zeros and the y-intercept of Rational Functions

Use the simplified form of the function.

To find x-intercepts (zeros), let $y = 0$ and solve for x, because the x-intercepts (zeros) are values of x that produce a y-value of 0.

To find the y-intercept, let $x = 0$ and solve for y, because the y-intercept is the point where the function intersects the y-axis.

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x^2 - 3x - 4}{x^2 - 16}$$

③ vertical asymptote(s): $x = -4$

② hole (point of discontinuity): $\left(4, \frac{5}{8}\right)$

④ horizontal asymptote: $y = 1$

⑤ x-intercept: $(-1, 0)$

⑥ y-intercept: $\left(0, \frac{1}{4}\right)$

① $f(x) = \frac{(x-4)(x+1)}{(x+4)(x-4)}$

$f(x) = \frac{x+1}{x+4}$

② hole where

$x - 4 = 0$

$+4 \quad +4$

$x = 4 \rightarrow (4, ?)$

$f(4) = \frac{4+1}{4+4} = \frac{5}{8}$

③ vert. asy. where $x + 4 = 0$

$-4 \quad -4$

$x = -4$ equation

④ $f(x) = \frac{x+1}{x+4}$

simp. $\frac{x}{x} = 1 \rightarrow y = 1$

⑤ $f(x) = \frac{x+1}{x+4}$

$(x+4) \cdot 0 = \frac{x+1}{x+4} \cdot (x+4)$

$0 = x + 1$

$-1 \quad -1$

$x = -1 \rightarrow (-1, 0)$ x-int.

⑥ $f(0) = \frac{0+1}{0+4}$

$y = \frac{1}{4} \rightarrow \left(0, \frac{1}{4}\right)$

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{2x + 1}{x - 3}$$

vertical asymptote(s): ____________________________

hole (point of discontinuity): _____________________

horizontal asymptote: _________________

x-intercept: ___________________

y-intercept: ___________________

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{2x+1}{x-3}$$

vertical asymptote(s): $x = 3$

hole (point of discontinuity): *none*

horizontal asymptote: $y = 2$

x-intercept: $(-\frac{1}{2}, 0)$

y-intercept: $\left(0, -\frac{1}{3}\right)$

$$f(x) = \frac{2x+1}{x-3}$$

vertical asymptote

$x - 3 = 0$

$x = 3$

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x+5}{x^2+5x}$$

vertical asymptote(s): ____________________________

hole (point of discontinuity): _____________________

horizontal asymptote: _________________

x-intercept: ___________________

y-intercept: ___________________

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x+5}{x^2+5x}$$

③ vertical asymptote(s): $x = 0$

② hole (point of discontinuity): $\left(-5, \frac{-1}{5}\right)$

④ horizontal asymptote: $y = 0$

⑤ x-intercept: none

⑥ y-intercept: none

① $f(x) = \frac{(x+5)}{x(x+5)}$

$f(x) = \frac{1}{x}$

② $x + 5 = 0$

$x = -5 \rightarrow \left(-5, \frac{-1}{5}\right)$

$f(-5) = \frac{1}{-5}$

③ $f(x) = \frac{1}{x} \rightarrow$ where $x = 0$

④ $\frac{1}{x} \rightarrow y = 0$

⑤ $f(x) = \frac{1}{x}$

$x \cdot 0 = \frac{1}{x} \cdot x$

$0 \neq 1 \rightarrow$ no x-int.

⑥ $f(0) = \frac{1}{0} \rightarrow$ undefined

no y-int.

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x-4}{x^2-4x+3}$$

vertical asymptote(s): ____________________________

hole (point of discontinuity): _____________________

horizontal asymptote: _________________

x-intercept: ____________________

y-intercept: ____________________

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x-4}{x^2-4x+3}$$

$$f(x) = \frac{x-4}{(x-3)(x-1)}$$

vertical asymptote(s): $x = 1, x = 3$

hole (point of discontinuity): *none*

horizontal asymptote: $y = 0$

x-intercept: $(4,0)$

y-intercept: $\left(0, -\frac{4}{3}\right)$

Objective: Find key characteristics of rational functions.

$$f(x) = \frac{x^2 - 9}{\sqrt{x + 5}}$$

① vertical asymptote(s): $x = -5$

② y-intercept: $\left(0, \frac{-9\sqrt{5}}{5}\right)$

① where $\sqrt{x+5} = 0$

$(\sqrt{x+5})^2 = (0)^2$

$x + 5 = 0$

$-5 \quad -5$

$x = -5$

equation

② $f(0) = \frac{0^2 - 9}{\sqrt{0 + 5}}$

$y = \frac{-9}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{-9\sqrt{5}}{5}$

Objective: Solve rational equations.

Solve the equation.

$$x - 4 = \frac{21}{x}$$

① LCD = x

② $x \cdot x - 4 \cdot x = \frac{21}{x} \cdot x$

③ solve.

$x^2 - 4x = 21$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$x - 7 = 0$ or $x + 3 = 0$

$x = 7$ ✓ $\quad x = -3$ ✓

④ check

solutions: $x = -3, 7$

Objective: Solve rational equations.

Solve the equation.

$$x = \frac{24}{x + 2}$$

Objective: Solve rational equations.

Solve the equation.

$$x = \frac{24}{x+2}$$

$$LCD = (x+2)$$

$$(x+2)\cdot x = \frac{24}{x+2}\cdot(x+2)$$

$$x^2 + 2x = 24$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$x+6 = 0 \ or \ \ x-4 = 0$$

$$x = -6 \ \ checks! \qquad x = 4 \ checks!$$

$$\boxed{solutions : x = -6, 4}$$

Objective: Solve rational equations.

Solve the equation.

$$\frac{6}{x} = \frac{18}{3x}$$

① LCD = $3 \cdot x$
= $3x$

② $3x \cdot \frac{6}{x} = \frac{18}{3x} \cdot 3x$

$18 = 18$ true

③ infinitely many real solutions
or
all real numbers

Objective: Solve rational equations.

Solve the equation.

$$\frac{2}{5x} = \frac{1}{10x}$$

LCD = 5 · 2 · x

= 10x

$$10x \cdot \frac{2}{5x} = \frac{1}{10x} \cdot 10x$$

$4 \neq 1$

$\emptyset$

Objective: Solve rational equations.

Solve the equation.

$$\frac{2}{5x} = \frac{1}{10x}$$

$LCD = 10x$

2

$$\cancel{10}\,\cancel{x} \cdot \frac{2}{\cancel{5}\,\cancel{x}} = \frac{1}{\cancel{10}\,\cancel{x}} \cdot \cancel{10}\,\cancel{x}$$

$4 \neq 1$

no solution

$\varnothing$