Objective: Find the inverse of a function from symbolic form

## Concept

How do we find the inverse of a function?

We interchange (switch) the $x$ and $y$ coordinates in all the points.

How many ordered pairs are there for the function $f(x) = 2x$?

There are an infinite number of ordered pairs.

If functions in symbolic form have an infinite number of ordered pairs, how can we find the inverse?

We can interchange (switch) the $x$ and $y$ variables in the symbolic form and then use algebra to solve for the $y$ variable.

# Objective: Find the inverse of a function from symbolic form

structure
$y = mx + b$

## Concept

**The inverse of a linear function is a linear function**. The domain of the function and its inverse will be all real numbers.

**Steps to Find the Inverse Function of a Linear Function from Symbolic Form**

1. Change the function notation to $y$.

2. Interchange (switch) the $x$ and $y$ variables. Do not move any numbers or other symbols.

3. Use algebra to solve for the $y$ variable.

4. Rewrite the $y$ variable using inverse notation.

Objective: Find the inverse of a function from symbolic form

Ex) ① Find the inverse of each function. State the domain of the inverse in set notation.

$$f(x) = -3x$$

linear $(y = mx + b)$

①

$y = -3x$

$x = -3y$

solve for $y$

$$\frac{1x}{-3} = \frac{-3y}{-3}$$

$y = -\frac{1}{3}x$

$$\boxed{f^{-1}(x) = -\frac{1}{3}x}$$

linear function

② domain of the inverse $\{x \mid -\infty < x < \infty\}$

"the set of all real numbers"

Objective: Find the inverse of a function from symbolic form

Ex) Find the inverse of each function. State the domain of the inverse in set notation.

$$g(x) = \frac{2}{3}x + 4$$

linear function

$y = mx + b$ structure

1

$y = \frac{2}{3}x + 4$

$x = \frac{2}{3}y + 4$

solve for y

$-4 \qquad -4$

$x - 4 = \frac{2}{3}y$

$\frac{3}{2}\cdot(x-4) = \frac{3}{2}\cdot\frac{2}{3}y$

$\frac{3}{2}\cdot x - \frac{3}{2}\cdot\frac{4}{1}$

$\frac{3}{2}x - 6 = y$

$\boxed{g^{-1}(x) = \frac{3}{2}x - 6}$

linear function

2 $\boxed{\text{domain of the inverse} \quad \{x \mid -\infty < x < \infty\}}$

Objective: Find the inverse of a function from symbolic form

## Concept

**The inverse of a quadratic function is a square root function.** To make the inverse a function, restrictions must be placed on the domain and range.

**Steps to Find the Inverse Function of a Quadratic Function from Symbolic Form**

1. Change the function notation to $y$.

2. Interchange (switch) the $x$ and $y$ variables. Do not move any numbers or other symbols.

3. Use algebra to solve for the $y$ variable. When using the square root property, only use the principle square root (positive value). This will make the inverse a function.

4. Determine the restricted domain. This limits the range to values in the set of real numbers.

5. Rewrite the $y$ variable using inverse notation. Write the restricted domain.

Objective: Find the inverse of a function from symbolic form

Ex) ① Find the inverse of each function. State the domain of the inverse in set notation.

$$f(x) = (x-4)^2 + 3$$ quadratic function

①

$y = (x-4)^2 + 3$

$x = (y-4)^2 + 3$

solve for y

$-3 \qquad -3$

$x - 3 = (y-4)^2$

principle square root

$\sqrt{x-3} = \sqrt{(y-4)^2}$

$\sqrt{x-3} = y - 4$

$+4 \qquad +4$

$\sqrt{x-3} + 4 = y$

$\boxed{f^{-1}(x) = \sqrt{x-3} + 4}$

② domain of the inverse

radicand must be nonnegative

$x - 3 \geq 0$

$+3 \qquad +3$

$x \geq 3$

$\boxed{\{x \mid x \geq 3\}}$

Captured on Fri Sep 27 2019 14:31:40

Objective: Find the inverse of a function from symbolic form

Ex) Find the inverse of each function. State the domain of the inverse in set notation.

$$d(x) = x^2 - 2$$

① 

$y = x^2 - 2$

$x = y^2 - 2$

solve for y

$+2 \qquad +2$

$x + 2 = y^2$

principle square root

$\sqrt{x+2} = \sqrt{y^2}$

$y = \sqrt{x+2}$

$\boxed{d^{-1}(x) = \sqrt{x+2}}$

② domain of the inverse

$x + 2 \geq 0$

$-2 \qquad -2$

$\boxed{\{x \mid x \geq -2\}} \leftarrow x \geq -2$

Objective: Find the inverse of a function from symbolic form

Concept

**The inverse of a cubic function is a cube root function.** Since the cube root of a negative number is a negative real number, no restrictions are needed to make the inverse a function.

**Steps to Find the Inverse Function of a Cubic Function from Symbolic Form**

1. Change the function notation to $y$.

2. Interchange (switch) the $x$ and $y$ variables. Do not move any numbers or other symbols.

3. Use algebra to solve for the $y$ variable.

4. Rewrite the $y$ variable using inverse notation.

Objective: Find the inverse of a function from symbolic form

Ex) Find the inverse of each function. State the domain of the inverse in set notation.

$$f(x) = 0.25x^3$$

1

$$y = 0.25x^3$$

$$y = \frac{1}{4}x^3$$

$$x = \frac{1}{4}y^3$$

solve for y

$$4 \cdot x = \frac{\cancel{4}^{1}}{1} \cdot \frac{1}{\cancel{4}_{1}} y^3$$

$$4x = y^3$$

cube root both sides

$$\sqrt[3]{4x} = \cancel{\sqrt[3]{y^3}}$$

$$y = \sqrt[3]{4x}$$

$$\boxed{f^{-1}(x) = \sqrt[3]{4x}}$$

2 domain of the inverse

★ no restrictions on cube roots

$$\boxed{\{x \mid -\infty < x < \infty\}}$$

"the set of all real numbers"

Objective: Find the inverse of a function from symbolic form

Ex) Find the inverse of each function. State the domain of the inverse in set notation.

$$d(x) = 2x^3 + 10$$

1

$y = 2x^3 + 10$

$x = 2y^3 + 10$

solve for y

$-10 \qquad -10$

$\frac{x-10}{2} = \frac{2y^3}{2}$

$\frac{1x}{2} - \frac{10}{2}$

$\frac{1}{2}x - 5 = y^3$

$\sqrt[3]{\frac{1}{2}x - 5} = \sqrt[3]{y^3}$

$y = \sqrt[3]{\frac{1}{2}x - 5}$

$\boxed{d^{-1}(x) = \sqrt[3]{\frac{1}{2}x - 5}}$

2 domain of the inverse

$\boxed{\{x \mid -\infty < x < \infty\}}$