Objective: Find the domain of a logarithmic function

Concept

A logarithm, $log_b a = n$ is defined for positive values of the argument, a. This is so the exponential form is defined.

Consider each of the following logarithms where the argument is positive, zero, and negative. Change each to its exponential form. For which arguments is the exponential form defined?

		Exponential Form	Circle One
argument is positive	$\log_3 9 = x$ $= 2$	$3^{x} = 9 \rightarrow 3^{2} = 9$	Defined/ Not Defined
argument is zero	$\log_3 0 = x$	$3^{x} \neq 0$	Defined/ Not Defined
argument is negative	$\log_3 -1 = x$	$3^{x} \neq -1$	Defined/ Not Defined

Objective: Find the domain of a logarithmic function

Concept

Since a logarithm, $log_b a = n$ is defined for positive values of the argument, a, this means the domain of a logarithmic function must be restricted to values of the independent variable, x, so that the argument is greater than 0.

Steps to Determining the Domain of a Logarithmic Function

1. Set the argument greater than 0.
2. Solve the inequality.

When Solving Linear Inequalities: Remember to change the inequality symbol if multiplying or dividing both sides by a negative number.

When Solving Quadratic Inequalities: Remember that the solutions to the related equation create intervals that must be tested to determine whether the values of x in each interval result in positive or negative values of the function.

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = log(9 - x) + 1$$

argument

① argument > 0

$9 - x > 0$ $\quad$ $-9 \qquad -9$

$-x > -9$

$\overline{-1} \qquad \overline{-1}$

$x < 9$

or

$9 - x > 0$

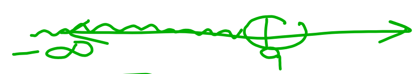
$+x \qquad +x$

$9 > x$

$x < 9$

② solve

* linear



③ $\boxed{\text{domain: } \{x \mid x < 9\} \quad (-\infty, 9)}$

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = ln\underbrace{(4x+7)}_{\text{argument}} + 8$$

1. $4x + 7 > 0$

   $\quad\; -7 \quad\; -7$

2. $\frac{4x}{4} > \frac{-7}{4}$

   $x > -\frac{7}{4}$

* linear

3. domain: $\{x \mid x > -\frac{7}{4}\}$

   $\left(-\frac{7}{4}, \infty\right)$

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = \log(64 - x^2) + 10$$

① $64 - x^2 > 0$

② solve. *quadratic

ⓐ zeros of $64 - x^2 = 0$

$+x^2 \quad +x^2$

$64 = x^2$

$\pm\sqrt{64} = \sqrt{\cancel{x^2}}$

$x = -8, 8$

ⓑ test $64 - x^2 > 0$

neg | pos | neg

$-8 \quad 0 \quad 8$

domain

③ $\boxed{\text{domain: } \{x \mid -8 < x < 8\} \quad (-8, 8)}$

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = ln(x^2 - 20)$$

① $x^2 - 20 > 0$

② ⓐ zeros $x^2 - 20 = 0$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm 2\sqrt{5}$

ⓑ test $x^2 - 20 \overset{?}{>} 0$

pos neg. pos

domain $-2\sqrt{5}$ 0 $2\sqrt{5}$ domain

③ $\boxed{\text{domain: } \{x \mid x < -2\sqrt{5} \text{ or } x > 2\sqrt{5}\} \quad (-\infty, -2\sqrt{5}) \cup (2\sqrt{5}, \infty)}$

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = log(x^2 + 10x + 25)$$

① $x^2 + 10x + 25 > 0$

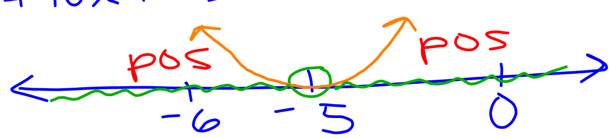
② ⓐ zeros $x^2 + 10x + 25 = 0$

$(x+5)(x+5) = 0$

$x + 5 = 0$ or $x + 5 = 0$

$x = -5$ $\quad$ $x = -5$

ⓑ test $x^2 + 10x + 25 \overset{?}{>} 0$



③ domain: $\{x \mid x < -5 \text{ or } x > -5\}$

$(-\infty, -5) \cup (-5, \infty)$

Objective: Find the domain of a logarithmic function

Ex) Find the domain of the function. Use set and interval notation.

$$g(x) = \ln(x^2 + 16) - 20$$

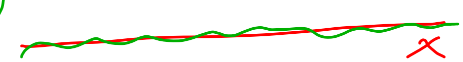
1 $x^2 + 16 > 0$

2 a

$$\begin{array}{rcl} x^2 + 16 & = & 0 \\ -16 & & -16 \\ \hline x^2 & = & -16 \\ \sqrt{\cancel{x^2}} & = & \pm\sqrt{-16} \\ x & = & \pm 4i \end{array}$$

$\sqrt{16}\cdot\sqrt{-1}$

b



3 $\boxed{\text{domain: } \{x \mid -\infty < x < \infty\} \quad (-\infty, \infty)}$

Objective: Find the domain of a logarithmic function

Closure

Kevin stated that the domain of $f(x) = ln(5 - x)$ is $x > 5$.
Test a value of x that is in this domain to determine whether Kevin is correct.
Explain why you believe Kevin is correct or incorrect.

Possible solution:
$x = 6 > 5$
$f(6) = ln(5 - 6) = ln(-1)$
Since there is no power of e that is equal to -1, Kevin is incorrect.
The correct domain is $x < 5$.