Objective: Graph Exponential Decay Functions and Find Key Features

Concept

An **exponential function** is a function of the form $f(x) = a(b)^{c(x-h)} + k$, **where** $b > 0$ **and** $b \neq 1$. The domain of every exponential function is all real numbers because the value of the exponent can be any real number.

What to include in the graph of an exponential function.

- **horizontal asymptote**; Note: for the function $f(x) = a(b)^{c(x-h)} + k$, the horizontal asymptote is $y = k$.
- **key points**, including the y-**intercept** and/or **zero** when reasonable
- **end behavior**

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There are two ways to write an exponential decay function of the form $f(x) = a(b)^{c(x-h)} + k$. One way is with a base value between 0 and 1 and a positive c value. The second way is with a base value greater than 1 and a negative c value.

Two Ways to Write the Exponential Decay Function

First Way: $\boldsymbol{f(x) = b^{-x+h}}$ **where** $\boldsymbol{b > 1}$

$$f(x) = b^{-(x-h)}$$

$$f(x) = (b^{-1})^{(x-h)}$$

Second Way: $\boldsymbol{f(x) = \left(\frac{1}{b}\right)^{x-h}}$ **where** $\boldsymbol{0 < \frac{1}{b} < 1}$

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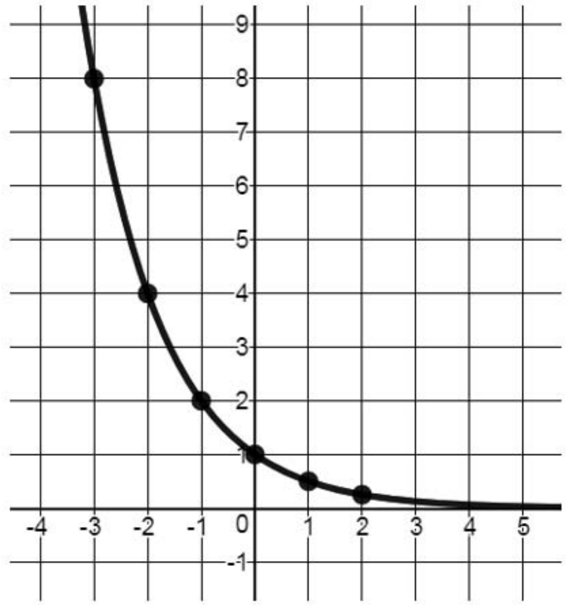
Concept

The function $\boldsymbol{f}(\boldsymbol{x}) = \left(\frac{1}{2}\right)^x$ is the parent function for all exponential functions with base $\frac{1}{2}$.

x	$f(x) = \left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\boldsymbol{f}(\boldsymbol{x}) = \left(\frac{\mathbf{1}}{\mathbf{2}}\right)^x$

or $f(x) = 2^{-x}$



Domain: the set of all real numbers; $\{x | -\infty < x < \infty\}$; $(-\infty, \infty)$
Range: $\{y | y > 0\}$; $(0, \infty)$
Horizontal Asymptote: $y = 0$
End Behavior: $as\ x \to -\infty, f(x) \to \infty$
; $as\ x \to \infty, f(x) \to 0$

Objective: Graph Exponential Decay Functions and Find Key Features

Ex) Graph the function. State the key features.

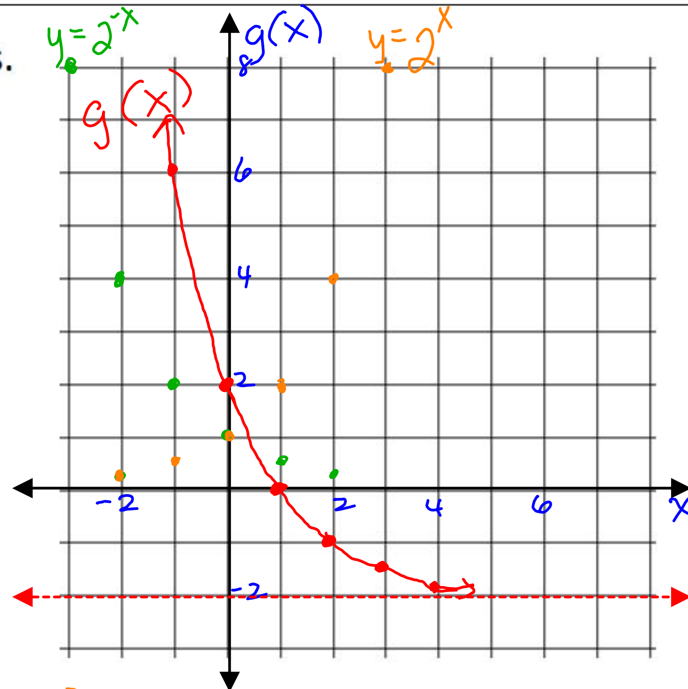
$$g(x) = \left(\frac{1}{2}\right)^{x-2} - 2$$

$a = 1$, $h = 2$, right, $k = -2$ down

y-int → $g(0)$

$g(0) = \left(\frac{1}{2}\right)^{0-2} - 2$

$= \left(\frac{1}{2}\right)^{-2} - 2 = 4 - 2 = 2$



Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$

(set notation) (interval notation)

Range: $\{y \mid y > -2\}$ / $(-2, \infty)$

(set notation) (interval notation)

y-intercept: $(0, 2)$

Horizontal Asymptote: $y = -2$

End Behavior: as $x \to -\infty$, $g(x) \to \infty$

as $x \to \infty$, $g(x) \to -2$

Objective: Graph Exponential Decay Functions and Find Key Features

Ex) Graph the function. State the key features.

$$g(x) = 2^{-x-3} + 5$$

$g(x) = 2^{-1(x+3)} + 5$ base

$g(x) = \left(\frac{1}{2}\right)^{x+3} + 5$ opp = h

$a = 1$, $\frac{1}{b} = -1$ y-axis refl. $h = -3$ left $k = 5$ up

uses $y = 2^x$ as parent

Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y > 5\}$ / $(5, \infty)$
(set notation) (interval notation)

y-intercept: $(0, 5\frac{1}{8})$

Horizontal Asymptote: $y = 5$

End Behavior: as $x \to -\infty$, $g(x) \to \infty$
as $x \to \infty$, $g(x) \to 5$

y-int $\to g(0)$

$g(0) = 2^{-1(0)-3} + 5$

$= 2^{-3} + 5$

$= \frac{1}{8} + 5 = 5\frac{1}{8}$

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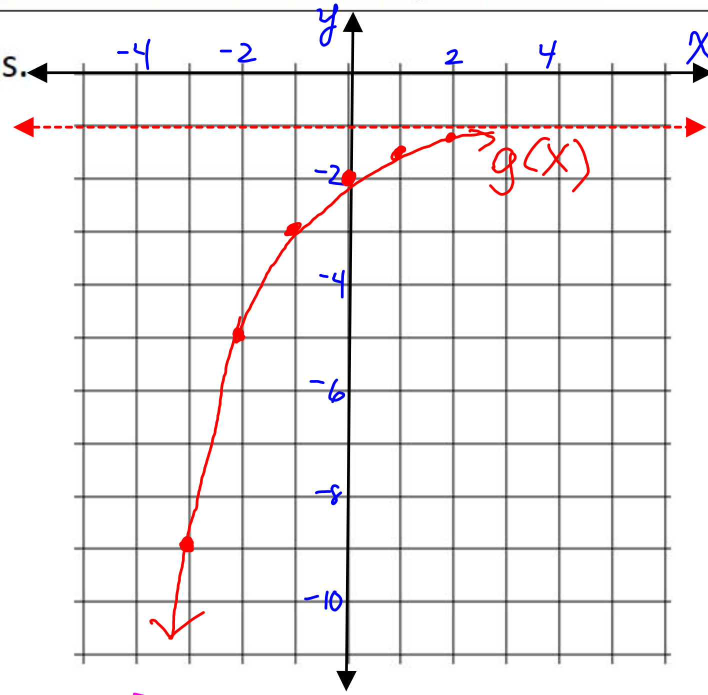
Objective: Graph Exponential Decay Functions and Find Key Features

Ex) Graph the function. State the key features.

$$g(x) = -2^{-x} - 1$$



$g(x) = -1 \cdot 2^{-1 \cdot x} - 1$

base

or $g(x) = -1 \cdot \left(\frac{1}{2}\right)^x - 1$

$a = -1$, $\frac{1}{b} = -1$, $h = 0$, $k = -1$

uses $y = 2^x$ as parent

x-axis refl

y-axis refl.

down

Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation) (interval notation)

Range: $\{y \mid y < -1\}$ / $(-\infty, -1)$
(set notation) (interval notation)

y-intercept: $(0, -2)$

Horizontal Asymptote: $y = -1$

End Behavior: as $x \to -\infty$, $g(x) \to -\infty$
as $x \to \infty$, $g(x) \to -1$

y-int → $g(0)$

$g(0) = -1 \cdot 2^{-1(0)} - 1$
$= -1 \cdot 2^0 - 1$
$= -1 \cdot 1 - 1$
$= -1 - 1$
$= -2$

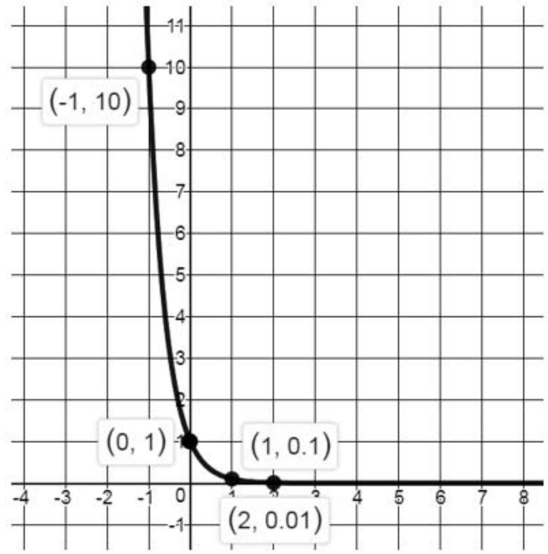
Objective: Graph Exponential Decay Functions and Find Key Features

Concept

The function $f(x) = \left(\frac{1}{10}\right)^x$ is the parent function for all exponential functions with base $\frac{1}{10}$.

x	$f(x) = \left(\frac{1}{10}\right)^x$
-2	$\left(\frac{1}{10}\right)^{-2} = 10^2 = 100$
-1	$\left(\frac{1}{10}\right)^{-1} = 10^1 = 10$
0	$\left(\frac{1}{10}\right)^0 = 1$
1	$\left(\frac{1}{10}\right)^1 = \frac{1}{10} = 0.1$
2	$\left(\frac{1}{10}\right)^2 = \frac{1}{100} = 0.01$

$f(x) = \left(\frac{1}{10}\right)^x$



Domain: the set of all real numbers; $\{x \mid -\infty < x < +\infty\}$; $(-\infty, +\infty)$
Range: $\{y \mid y > 0\}$; $(0, +\infty)$
Horizontal Asymptote: $y = 0$
End Behavior: $as\ x \to -\infty, f(x) \to +\infty$;
$as\ x \to +\infty, f(x) \to 0$

Objective: Graph Exponential Decay Functions and Find Key Features

Ex) Graph the function. State the key features.

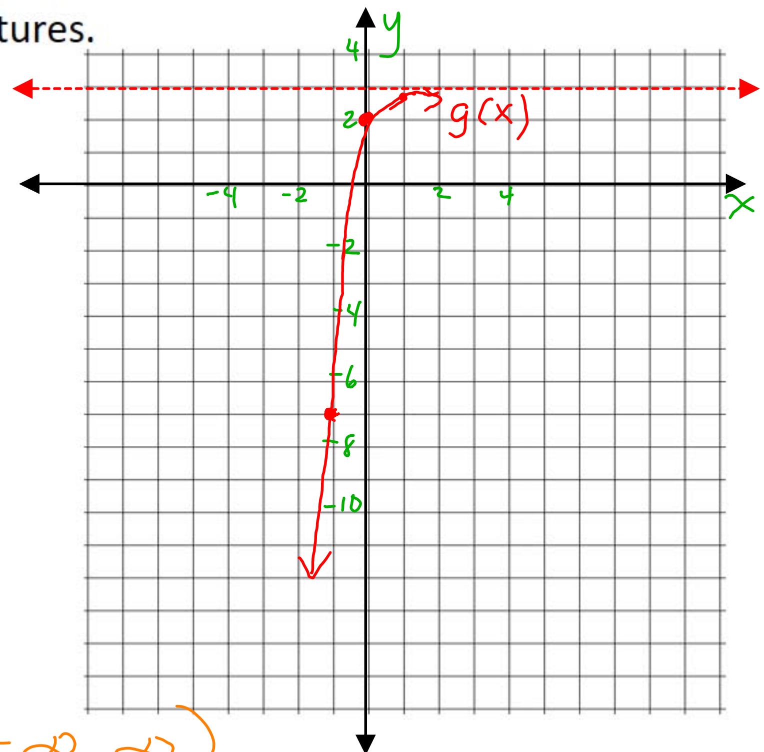
$$g(x) = -\left(\frac{1}{10}\right)^x + 3$$

10^x

$g(x) = -10^{-x} + 3$

$a = -1$, $h = 0$, $k = 3$

x-axis refl.   up



Domain: $\{x \mid -\infty < x < \infty\}$ / $(-\infty, \infty)$
(set notation)   (interval notation)

Range: $\{y \mid y < 3\}$ / $(-\infty, 3)$
(set notation)   (interval notation)

y-intercept: $(0, 2)$

Horizontal Asymptote: $y = 3$

End Behavior: as $x \to -\infty$, $g(x) \to -\infty$
as $x \to \infty$, $g(x) \to 3$

y-int $\to g(0)$

$g(0) = -\left(\frac{1}{10}\right)^0 + 3$

$= -1 + 3 = 2$