Objective: Graph square root functions using transformations.

Concept

For square root functions $f(x) = a\sqrt{x-h}+k$ and $g(x) = \sqrt{\frac{1}{b}(x-h)}+k$

the transformations are:

translation h units left or right

translation k units up or down

reflection across the x-axis when $a < 0$

vertical stretch when $|a| > 1$

vertical compression when $|a| < 1$

reflection across the y-axis when $b < 0$

horizontal stretch when $|b| > 1$ (i.e. if $\frac{1}{b} = \frac{1}{2} \rightarrow b = 2$)

horizontal compression when $|b| < 1$ (i.e. if $\frac{1}{b} = 2 \rightarrow b = \frac{1}{2}$)

Objective: Graph square root functions using transformations.

Ex) For each function, describe the effect of each parameter on the parent function $f(x) = \sqrt{x}$.

$$g(x) = -2\sqrt{x+3} - 1$$

a, opp = h, k

$a = -2$ x-axis ref.

$|a| = |-2| = 2 > 1$ vert. stretch

$h = -3$ left 3

$k = -1$ down 1

The effects of the parameters on the parent function $f(x) = \sqrt{x}$ are an x-axis reflection, a vertical stretch by a factor of 2 and a translation left 3 units and down 1 unit.

$$g(x) = \sqrt{-3(x-1)} + 2$$

b, opp = h, k

$\frac{1}{b} = -3 \rightarrow b = -\frac{1}{3}$ y-axis refl.

$|b| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$ hor. comp.

$h = 1$ right 1

$k = 2$ up 2

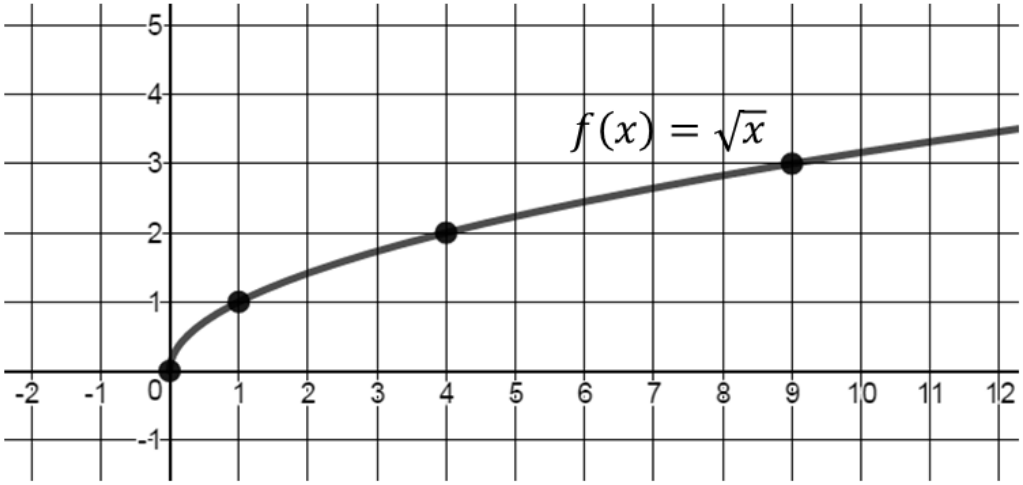
The effects of the parameters on the parent function $f(x) = \sqrt{x}$ are a y-axis reflection, a horizontal compression by a factor of $\frac{1}{3}$ and a translation right 1 unit and up 2 units.

Objective: Graph square root functions using transformations.

Concept

To graph square root functions using transformations we must know the graph of the parent function $f(x) = \sqrt{x}$.

x	$f(x) = \sqrt{x}$
0	0
1	1
4	2
9	3



Domain: $\{x|x \geq 0\}$; $[0, \infty)$ Range: $\{y|y \geq 0\}$; $[0, \infty)$

End Behavior: $as\ x \to +\infty, f(x) \to +\infty$

Note: End Behavior of a square root function is only described on the end where x goes to positive or negative infinity.

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Two Procedures to Graph a Square Root Function Using Transformations

One Option:

1. Translate the function horizontally and vertically. The end point $(0,0)$ will not be affected by step 2.
2. Stretch/Compress and Reflect all points other than the endpoint. For a parameter of a the line of reflection is now the horizontal line containing the translated endpoint. For a parameter of $\frac{1}{b}$ the line of reflection is now the vertical line containing the translated endpoint.

Another Option:

1. Stretch/Compress and Reflect all points (the endpoint $(0,0)$ is not affected.) The line of reflection is the x-axis for a parameter of a, or the y-axis for a parameter of $\frac{1}{b}$.
2. Translate the transformed function horizontally and vertically.

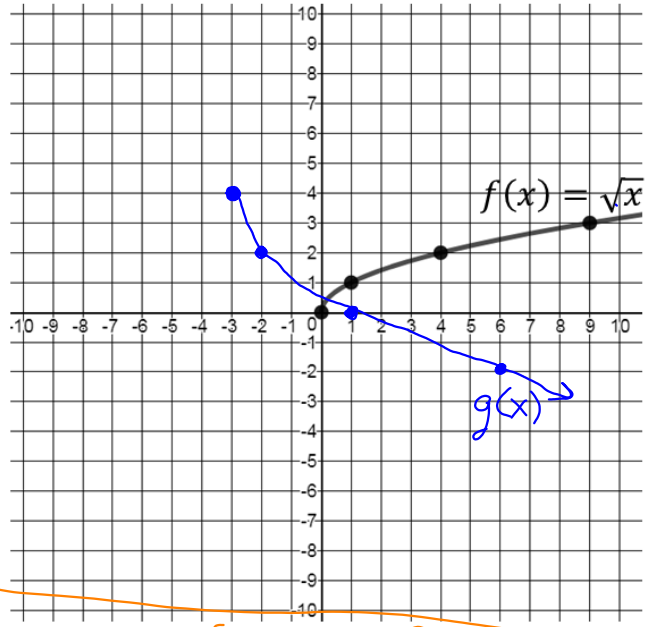
Objective: Graph square root functions using transformations.

Ex) Graph each function using transformations. Describe the domain and range using set and interval notation. Describe the end behavior.

$$g(x) = -2\sqrt{x+3} + 4$$

② $a = -2$ x-axis refl.
$|a| = |-2| = 2 > 1$ vert. stretch factor = 2

① $h = -3$ left 3
$k = 4$ up 4



Domain: $\{x \mid x \geq -3\}$ set
$[-3, \infty)$ interval

Range: $\{y \mid y \leq 4\}$ set
$(-\infty, 4]$ interval

as $x \to \infty$, $g(x) \to -\infty$
end behavior

Objective: Graph square root functions using transformations.

Ex) Graph each function using transformations. Describe the domain and range using set and interval notation. Describe the end behavior.

$$g(x) = \sqrt{\frac{1}{3}(x-1)} - 2$$

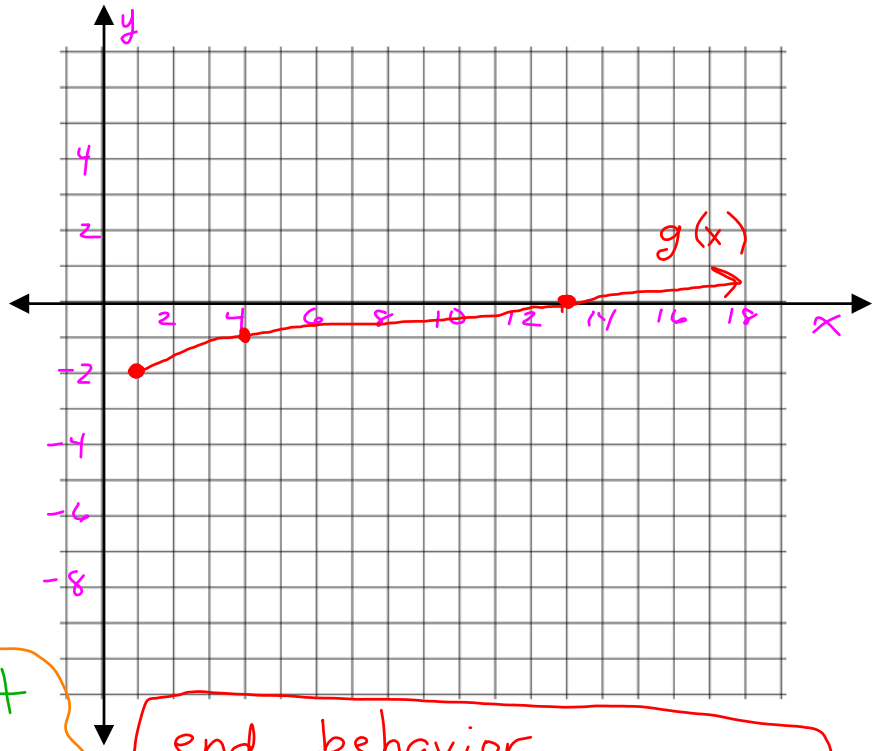
$\frac{1}{b}$

$\frac{1}{b} = \frac{1}{3} \rightarrow b = 3$

$|b| = |3| = 3 > 1$ hor. stretch

$h = 1$ right

$k = -2$ down 2



Domain: $\{x \mid x \geq 1\}$ set

$[1, \infty)$ interval

Range: $\{y \mid y \geq -2\}$ set

$[-2, \infty)$ interval

end behavior

as $x \rightarrow +\infty$, $g(x) \rightarrow +\infty$

Objective: Graph square root functions using transformations.

Ex) Graph each function using transformations. Describe the domain and range using set and interval notation. Describe the end behavior.

$$g(x) = \sqrt{-2(x+4)} - 5$$

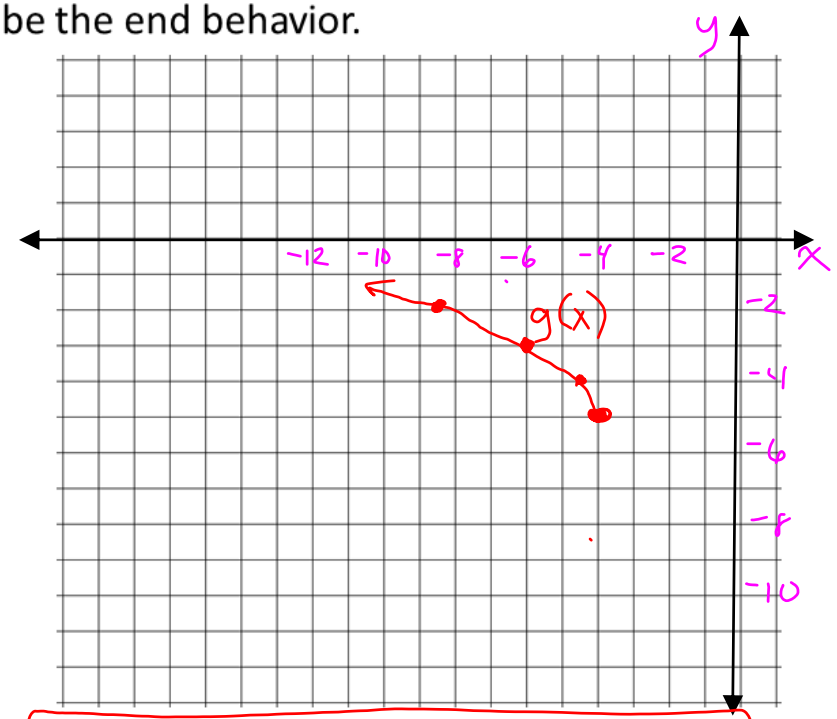
$\frac{1}{b}$

$\frac{1}{b} = -2 \rightarrow b = -\frac{1}{2}$ y-axis refl.

$|b| = \left|\frac{-1}{2}\right| = \frac{1}{2} < 1$ hor. comp.

$h = -4$ left 4

$k = -5$ down 5



Domain: $\{x \mid x \leq -4\}$

$(-\infty, -4]$

Range $\{y \mid y \geq -5\}$

$[-5, \infty)$

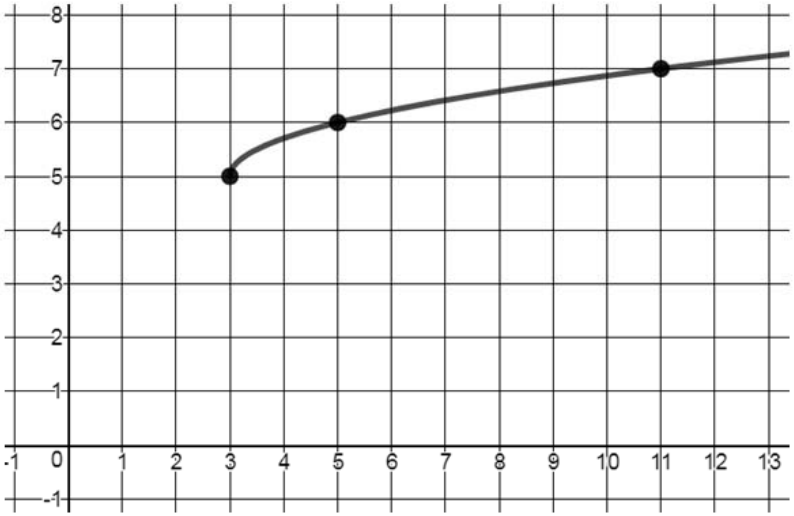
end behavior

as $x \rightarrow -\infty$, $g(x) \rightarrow +\infty$

Objective: Graph square root functions using transformations.

Closure

David graphed the function $h(x) = 2\sqrt{x-3} + 5$ using transformations. Explain David's error.



David graphed a horizontal stretch by a factor of 2 instead of a vertical stretch by a factor of 2.