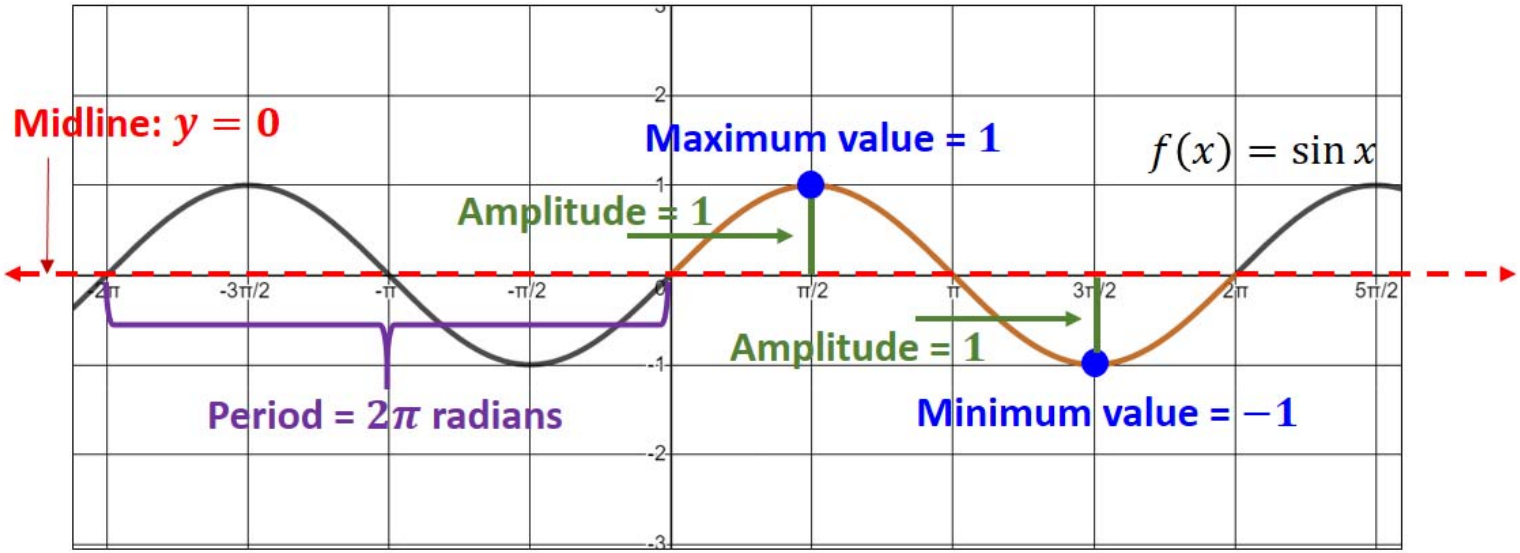
Objective: Graph sine and cosine with period changes and phase shifts.

Concept

The **period** of a sine or cosine function **is the length of one cycle** of the function. The **amplitude** of the sine function is **half the distance from its maximum to its minimum**. The amplitude is **also the distance from the midline to a maximum or minimum**. The **midline** of the sine function is the **horizontal line halfway between the function's maximum and minimum values**.



$$\text{Amplitude} = \frac{max - min}{2} = \frac{1-(-1)}{2} = \frac{2}{2} = 1$$

Objective: Graph sine and cosine with period changes and phase shifts.

Concept

The transformations you have learned can be applied to the sine and cosine functions as well.

A **horizontal translation is called a phase shift** for a trigonometric function. **When there is no horizontal translation, the phase shift is 0 radians.**

A **horizontal stretch/compression will change the period** to something other than 2π radians.

A **vertical stretch/compression will change the amplitude** to something other than 1.

A **vertical translation will change the midline** to something other than $y = 0$.

Note: While there can also be reflections across the x-axis and y-axis, these do not change the values of the period or the amplitude or the equation of the midline.

Objective: Graph sine and cosine with period changes and phase shifts.

Concept

For $g(x) = a \cdot \sin\left(\frac{1}{b}x - c\right) + k$ or $g(x) = a \cdot \cos\left(\frac{1}{b}x - c\right) + k$

Period of a sine or cosine function: $P = \frac{2\pi}{\left|\frac{1}{b}\right|} = 2\pi \cdot |b|$

Amplitude of a sine or cosine function: $A = |a|$

Midline of a sine or cosine function: $y = k$

Phase Shift of a sine or cosine function: $\frac{c}{\frac{1}{b}} = c \cdot b$

To graph a sine or cosine function with the above form:

1. Determine the period, amplitude, midline, and phase shift.
2. **The scale of $\frac{\pi}{4}$ radians is suggested for the x-axis. If this scale won't work, use the strategy of dividing the period by 4 to determine another possible scale for the x-axis.**
3. Mark the points at the beginning and end of the period. Fill in the middle with the other key points. Extend the cycle as needed to the left and right.
4. Draw a smooth curve.

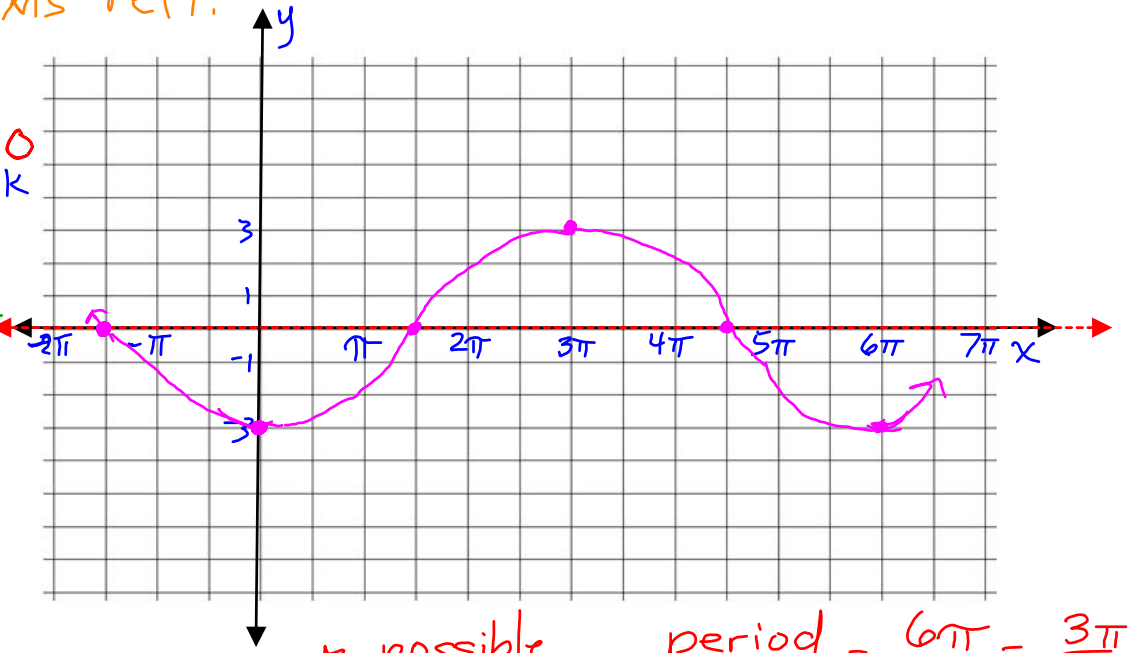
Objective: Graph sine and cosine with period changes and phase shifts.

Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

$g(x) = -3\cos\frac{x}{3}$ → x-axis refl.

$g(x) = -3\cos\left(\frac{1}{3}x - 0\right) + 0$

a, $\frac{1}{b}$, OPP=c, k



Period = $\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$

Amplitude = $|-3| = 3$

Midline: $y = 0$

* possible scale = $\frac{\text{period}}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$

Phase Shift = $\frac{0}{\frac{1}{3}} = 0$ radians or $\frac{\pi}{2}$ *

Objective: Graph sine and cosine with period changes and phase shifts.

Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

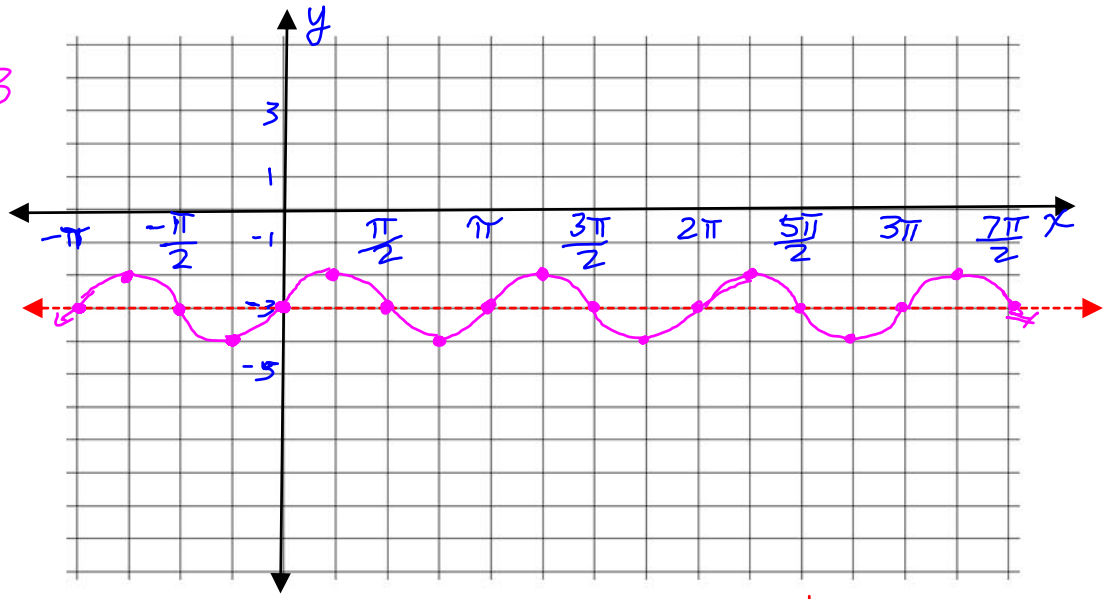
$g(x) = \sin 2x - 3$

$g(x) = \underset{a}{1} \cdot \sin(\underset{b}{2}x - \underset{\text{Opp.} = c}{0}) \underset{k}{-3}$

Period = $\frac{2\pi}{2} = \pi$

Amplitude = $|1| = 1$

Midline: $y = -3$



Phase Shift = $\frac{0}{2} = 0$ radians

* possible scale $= \frac{\pi}{4}$ or $\frac{\pi}{8}$

Objective: Graph sine and cosine with period changes and phase shifts.

Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

$$g(x) = \frac{2}{3}\sin\left(x - \frac{\pi}{2}\right)$$

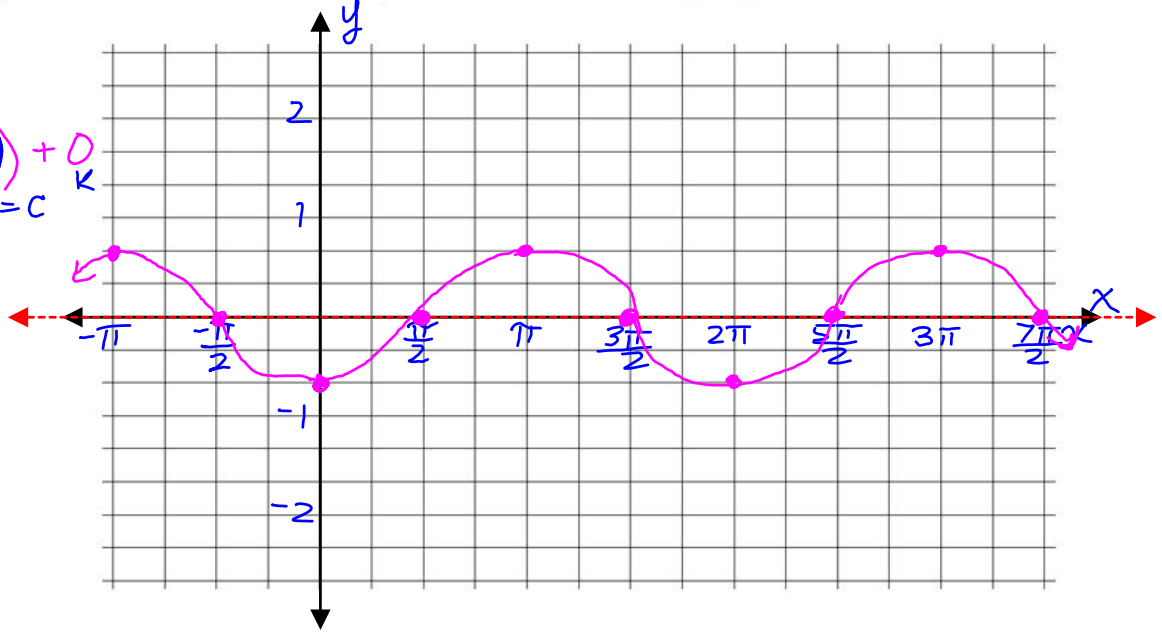
$g(x) = \frac{2}{3}\sin\left(1 \cdot x - \frac{\pi}{2}\right) + 0$

a, b, opp = c, k

Period = $\frac{2\pi}{1} = 2\pi$

Amplitude = $\left|\frac{2}{3}\right| = \frac{2}{3}$

Midline: $y = 0$



Phase Shift = $\frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$ (right $\frac{\pi}{2}$)

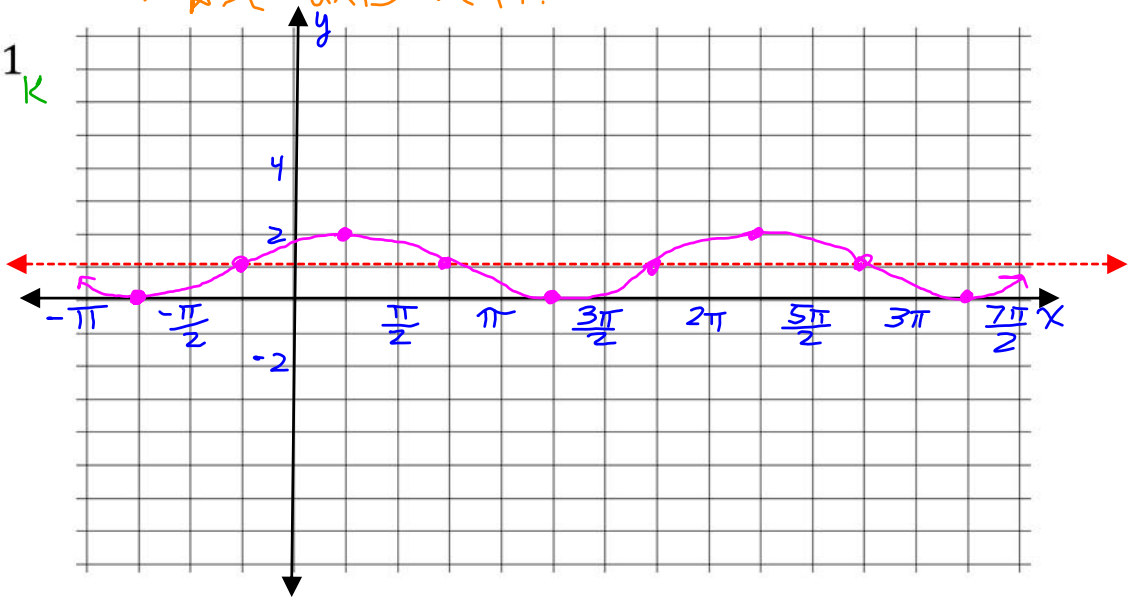
Objective: Graph sine and cosine with period changes and phase shifts.

Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

$$g(x) = -\cos\left(1x + \frac{3\pi}{4}\right) + 1$$

a = -1, b = 1, opp = c, k

* x-axis refl.



Period = $\frac{2\pi}{1} = 2\pi$

Amplitude = $|-1| = 1$

Midline: $y = 1$

Phase Shift = $\frac{-\frac{3\pi}{4}}{1} = -\frac{3\pi}{4}$ (left $\frac{3\pi}{4}$)

Objective: Graph sine and cosine with period changes and phase shifts.

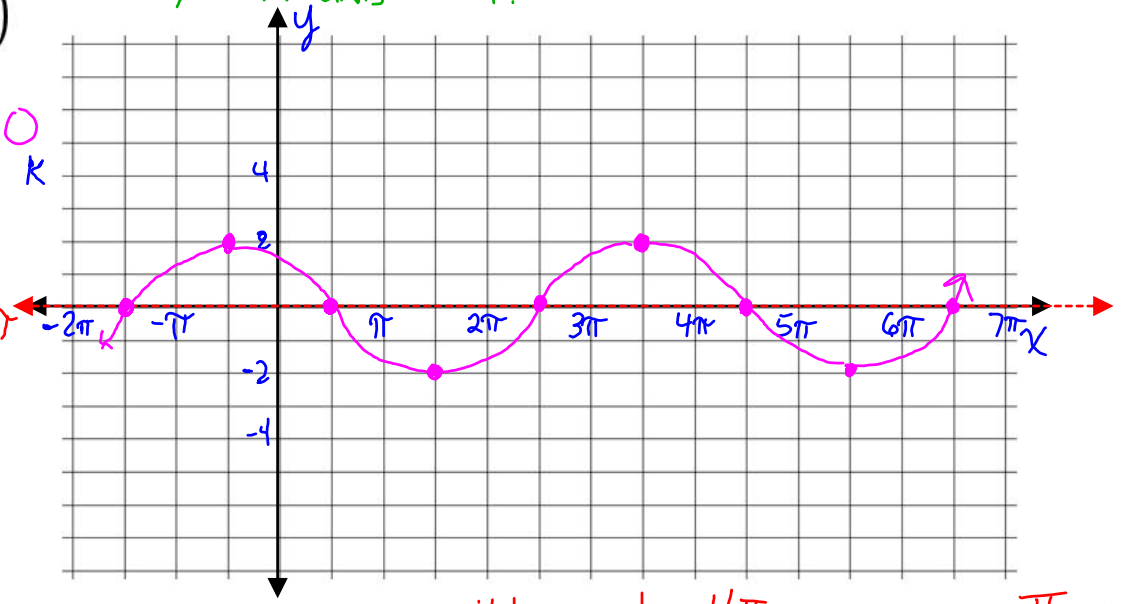
Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

$$g(x) = -2\sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$$

★ x-axis refl.

$g(x) = -2\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 0$

a, $\frac{1}{b}$, opp = c, k



Period = $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Amplitude = $|-2| = 2$

Midline: $y = 0$

★ possible scale $\frac{4\pi}{4} = \pi$ or $\frac{\pi}{2}$ ★

Phase Shift = $\frac{\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{4} \cdot \frac{2}{1} = \frac{\pi}{2}$ (right $\frac{\pi}{2}$)

Objective: Graph sine and cosine with period changes and phase shifts.

Ex) Find the period, amplitude, midline, and phase shift. Then graph the function.

$$g(x) = \frac{4}{3}\cos(4x + \pi) + 1$$

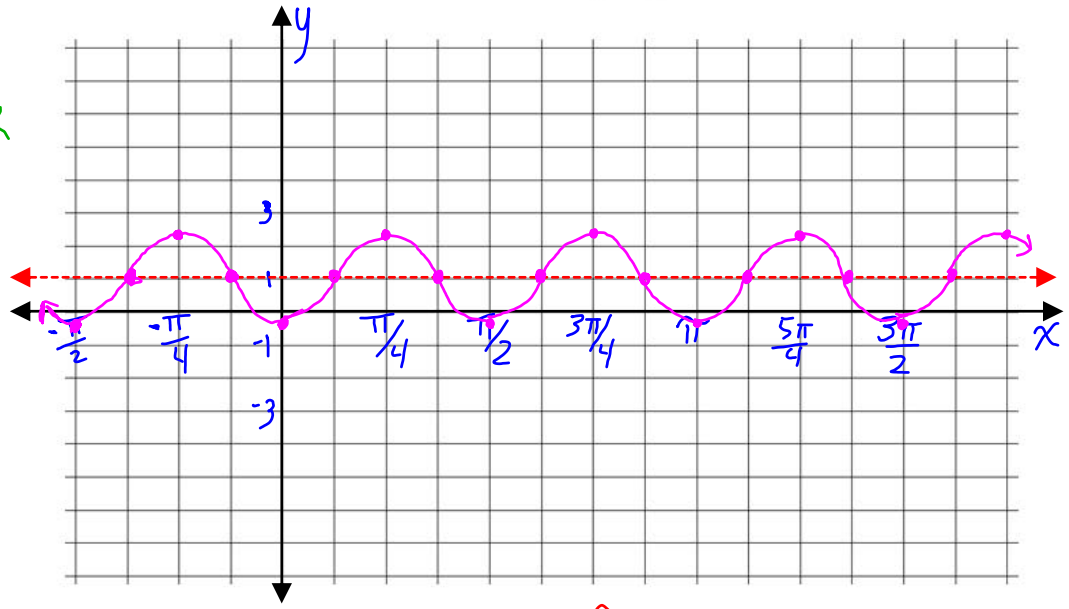
$a = \frac{4}{3}$ $\quad \frac{1}{b}$ $\quad$ opp=c $\quad$ K

Period = $\frac{2\pi}{4} = \frac{\pi}{2}$

Amplitude = $\left|\frac{4}{3}\right| = \frac{4}{3}$

Midline: $y = 1$

Phase Shift = $\frac{-\pi}{4}$ (left $\frac{\pi}{4}$)



* possible scale = $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$ or $\frac{\pi}{4}$

Objective: Graph sine and cosine with period changes and phase shifts.

Closure

For the function $f(x) = \frac{5}{4}\cos\left(\frac{x}{4} - \frac{\pi}{2}\right) - 3$, explain why the phase shift isn't $\frac{\pi}{2}$ radians. Then determine the correct phase shift.

The phase shift for the function $f(x) = \frac{5}{4}\cos\left(\frac{x}{4} - \frac{\pi}{2}\right) - 3$ isn't $\frac{\pi}{2}$ radians because the phase shift is found by dividing this value by $\frac{1}{4}$. The correct phase shift is $\frac{\frac{\pi}{2}}{\frac{1}{4}}$ which is 2π radians.