Objective: Solve square root equations algebraically.

Concept

A Radical Equation contains a variable within a radical or a variable raised to a non-integer rational exponent.

Examples

$\sqrt{x-4}=7$ $(x+5)^{\frac{1}{2}}=9$ $\sqrt[3]{x^2-5}=2$

Non-Examples

$\sqrt{x}+9$ (no equal sign)

$x-12=6$ (no radical or non-integer rational exponent)

$(x+4)^{\frac{1}{2}}$ (no equal sign)

Objective: Solve square root equations algebraically.

Concept

Steps to Solve a Radical Equation

1. **Isolate the radical expression.** If the equation contains more than one radical expression, choose one to isolate.
2. **Raise both sides of the equation to the appropriate power** so the isolated root and power cancel.
3. **Solve the resulting equation.** Be aware of whether the equation is linear or quadratic.
4. **Check for Extraneous Solutions** and then write the final solution set.

Radical equations can have **extraneous solutions**:

1. Solutions that fail to make the left side and right side of the equation equal.

2. Solutions that are imaginary or create imaginary values when substituted into the original equation.

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

① $\sqrt{x^2-5}=2$

② $(\sqrt{x^2-5})^2=(2)^2$

③ $x^2-5=4$

$+5 \quad +5$

$x^2=9$

$\sqrt{x^2}=\pm\sqrt{9}$

$x=-3,3$

④ check $x=-3$; $\sqrt{(-3)^2-5} \stackrel{?}{=} 2$

$\sqrt{9-5}$

$\sqrt{4}=2$

$2=2$ ✓

check $x=3$ $\sqrt{(3)^2-5} \stackrel{?}{=} 2$

$\sqrt{9-5}$

$\sqrt{4}=2$

$2=2$ ✓

solutions: $x=-3,3$

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

(1) $\sqrt{x^2+1} = 7$

(2) $(\sqrt{x^2+1})^2 = (7)^2$

(3) $x^2 + 1 = 49$
$-1 \quad -1$

$x^2 = 48$

$\sqrt{x^2} = \pm\sqrt{48}$
$\sqrt{16}\cdot\sqrt{3}$

$x = -4\sqrt{3}, 4\sqrt{3}$

(4) check: $x = -4\sqrt{3}$; $\sqrt{(-4\sqrt{3})^2+1} \stackrel{?}{=} 7$
$16\cdot 3$
$48+1$
$\sqrt{49} = 7$
$7 = 7\checkmark$

solutions:
$x = -4\sqrt{3}, 4\sqrt{3}$

$x = 4\sqrt{3}$; $\sqrt{(4\sqrt{3})^2+1} \stackrel{?}{=} 7$
$16\cdot 3$

$\sqrt{49} = 7$
$7 = 7\checkmark$

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

① $(x-1)^{\frac{1}{2}} = 16$

same as: $\sqrt{x-1} = 16$

② $(\sqrt{x-1})^2 = (16)^2$

③ $x - 1 = 256$

$+1 \quad +1$

$x = 257$

④ check $\sqrt{257-1} \stackrel{?}{=} 16$

$\sqrt{256} = 16$

$16 = 16$ ✓

solution: $x = 257$

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

$$2 + \sqrt{x+10} = x$$

① $-2 \quad -2$

$\sqrt{x+10} = x-2$

② $(\sqrt{x+10})^2 = (x-2)^2$

$(x-2)(x-2)$

$x+10 = x^2 - 4x + 4$

③ quadratic standard form

$-x \quad -10 \quad -x \quad -10$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x-6=0 \quad +6 \quad +6$

$x = 6$

$x+1=0 \quad -1 \quad -1$

$x = -1$ extraneous

④ check: $x = 6$

$2 + \sqrt{6+10} \stackrel{?}{=} 6$

$2 + \sqrt{16}$

$2+4 = 6$ ✓

solution

$x = 6$

check: $x = -1$

$2 + \sqrt{-1+10} \stackrel{?}{=} -1$

$2 + \sqrt{9}$

$2 + 3 \neq -1$

$5 \neq -1$

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

$$\sqrt{x-2} - \frac{3}{\sqrt{x-2}} = 0$$

① $+\frac{3}{\sqrt{x-2}} \quad +\frac{3}{\sqrt{x-2}}$

② multiply by the LCD

LCD $= \sqrt{x-2}$

$\sqrt{x-2} \cdot \frac{\sqrt{x-2}}{1} = \frac{3}{\sqrt{x-2}} \cdot \frac{\sqrt{x-2}}{1}$

$(\sqrt{x-2})^2 = 3$

③ $x - 2 = 3$

$+2 \quad +2$

$x = 5$

④ check $x = 5$; $\sqrt{5-2} - \frac{3}{\sqrt{5-2}} \stackrel{?}{=} 0$

$\sqrt{3} - \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\sqrt{3} - \frac{3\sqrt{3}}{3}$

$\sqrt{3} - \sqrt{3} = 0$ ✓

solution $x = 5$

Objective: Solve square root equations algebraically.

Ex) Solve the equation.

$$(x+6)^{\frac{1}{2}} - (2x-4)^{\frac{1}{2}} = 0$$

same as: $\sqrt{x+6} - \sqrt{2x-4} = 0$

① $+\sqrt{2x-4} \quad +\sqrt{2x-4}$

$\sqrt{x+6} = \sqrt{2x-4}$

② $\left(\sqrt{x+6}\right)^2 = \left(\sqrt{2x-4}\right)^2$

③ $x+6 = 2x-4$

$-x+4 \quad -x+4$

$10 = x$

④ check:

$x = 10$; $\sqrt{10+6} - \sqrt{2(10)-4} \stackrel{?}{=} 0$

$\sqrt{16} - \sqrt{16} = 0$

$4 - 4 = 0$

$0 = 0$ ✓

Solution: $x = 10$

Objective: Solve square root equations algebraically.

Closure

Jessica solved a square root equation. Her work is shown. Is her solution correct? Explain your reasoning.

$$\sqrt{x^2+29}=2$$

$$\left(\sqrt{x^2+29}\right)^2=(2)^2$$

$$x^2+29=4$$

$$x^2==-25$$

$$\sqrt{x^2}=\pm\sqrt{-25}$$

$$x=\pm 5i$$

$$\boxed{solution: x=-5i, 5i}$$

Jessica's solution is incorrect. Solutions that are imaginary are extraneous for radical equations. She should have concluded there is no solution to the equation.