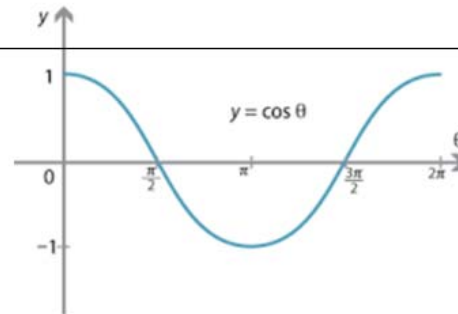


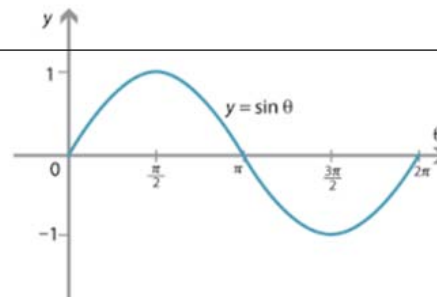
## Objective: Solve Trigonometric Equations

### Concept

For the function  $\cos x = y$ : the domain,  $x$ , which is all angle measures, is the set of all real numbers and the range,  $y$ , is the values of cosine in the interval  $[-1, 1]$ .



For the function  $\sin x = y$ : the domain,  $x$ , which is all angle measures, is the set of all real numbers and the range,  $y$ , is the values of sine in the interval  $[-1, 1]$ .



Objective: Solve Trigonometric Equations

Steps to Solve a Trigonometric Equation

**Linear Structure**

1. Combine like terms.
2. Isolate the trigonometric function.
3. Find the angle measure(s) that correspond to the function value.

**Quadratic Structure**

1. Write the equation in standard form.
2. Use a Quadratic Strategy (square root property, factoring, quadratic formula) to solve for the trigonometric function values.
3. Find the angle measure(s) that correspond to the function value(s).

Objective: Solve Trigonometric Equations

Ex) Solve each equation over the interval  $[0, 2\pi)$ .

① solve

★ linear structure

$$\begin{array}{r} \sin x + \sqrt{2} = -\sin x \\ +\sin x \quad \quad +\sin x \\ \hline 2\sin x + \sqrt{2} = 0 \\ \quad \quad -\sqrt{2} \quad -\sqrt{2} \\ \hline \sin x = -\frac{\sqrt{2}}{2} \\ \sin x = -\frac{\sqrt{2}}{2} \end{array}$$

② use the Unit Circle

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

Objective: Solve Trigonometric Equations

Ex) Solve each equation over the interval  $[0, 2\pi)$ .

$$3\tan^2 x - 1 = 0$$

$$\frac{3\tan^2 x}{3} = \frac{1}{3}$$

$$\tan^2 x = \frac{1}{3}$$

$$\sqrt{\tan^2 x} = \pm \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{\sqrt{1}}{\sqrt{3}} = \pm \frac{1}{\sqrt{3}}$$

$$\frac{\sin x}{\cos x} = \frac{\pm \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \pm \frac{1}{\sqrt{3}}$$

↑ QII  
QIV    ↑ QI  
QIII

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

\* quadratic structure  
① solve

② use unit circle

Objective: Solve Trigonometric Equations

Ex) Solve each equation over the interval  $[0, 2\pi)$ .

\* quadratic structure

① standard form

$$\begin{aligned} \cot x \cos^2 x &= 2 \cot x \\ &\quad - 2 \cot x \quad - 2 \cot x \\ \hline \cot x \cos^2 x - 2 \cot x &= 0 \\ \cot x (\cos^2 x - 2) &= 0 \end{aligned}$$

$$\cot x = 0 \quad \text{or} \quad \frac{\cos^2 x - 2}{+2 \quad +2} = 0$$

$$\downarrow$$

$$\frac{\cos x}{\sin x} = \frac{0}{1}$$

② Unit Circle

or

$$\frac{\cos x}{\sin x} = \frac{0}{-1}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 x = 2$$

$$\sqrt{\cos^2 x} = \pm \sqrt{2}$$

$$\cos x = -\sqrt{2}, \sqrt{2}$$

invalid  $\approx -1.4, 1.4$   
range for cosine  $[-1, 1]$

Objective: Solve Trigonometric Equations

Ex) Solve each equation over the interval  $[0, 2\pi)$ .

\*quadratic structure

$$2 \sin^2 x - \sin x = 1$$

-1 -1  
—————

① standard form

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

factor

$$2x^2 - x - 1$$

$$(2x + 1)(x - 1)$$

$$2 \sin x + 1 = 0 \text{ or } \sin x - 1 = 0$$

$$\frac{2 \sin x}{2} = \frac{-1}{2}$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

② unit circle

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Objective: Solve Trigonometric Equations

Closure

Explain why the equation  $\sin x = 3$  has no solution.

The equation  $\sin x = 3$  has no solution because 3 is outside of the range values of the sine function, which is from -1 to 1.

