Objective: Solve right triangles.

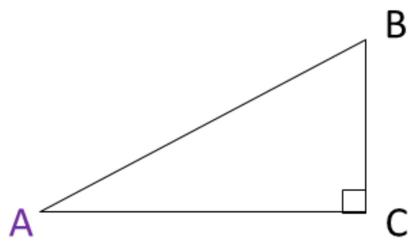
Concept

A **trigonometric ratio is a ratio of two sides of a right triangle**. The three primary trigonometric ratios are sine, cosine, and tangent.

$$\sin A = \frac{\text{length of side } \textbf{opposite} \text{ angle } \angle A}{\text{length of } \textbf{hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of side } \textbf{adjacent} \text{ to angle } \angle A}{\text{length of } \textbf{hypotenuse}} = \frac{AC}{AB}$$

$$\tan A = \frac{\text{length of side } \textbf{opposite} \text{ angle } \angle A}{\text{length of side } \textbf{adjacent} \text{ to angle } \angle A} = \frac{BC}{AC}$$



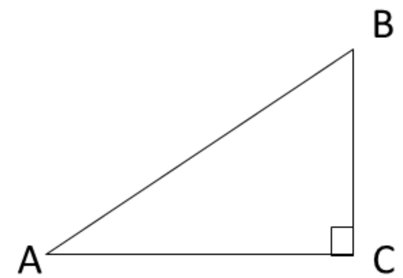
SOHCAHTOA

Objective: Solve right triangles.

Concept

In general, if you know the lengths of two sides of a right triangle you can use the corresponding inverse trigonometric function to find the acute angle. This is expressed mathematically in the statements below.

Trigonometric Ratios	Inverse Trigonometric Ratios
$\sin A = \frac{opposite}{hypotenuse}$	$m\angle A = \sin^{-1}\left(\frac{opposite}{hypotenuse}\right)$
$\cos A = \frac{adjacent}{hypotenuse}$	$m\angle A = \cos^{-1}\left(\frac{adjacent}{hypotenuse}\right)$
$\tan A = \frac{opposite}{adjacent}$	$m\angle A = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$



Note: sin^{-1} is read as "*sine inverse*"

$sin^{-1}\left(\frac{opposite}{hypotenuse}\right)$ represents the angle measure with the sine ratio $\frac{opposite}{hypotenuse}$.

Objective: Solve right triangles.

Concept

To solve a right triangle means finding the lengths of all its sides and the measures of all its angles. To solve a right triangle you need to know two side lengths or one side length and an acute angle measure. Always use exact measures, when available, to solve a triangle.

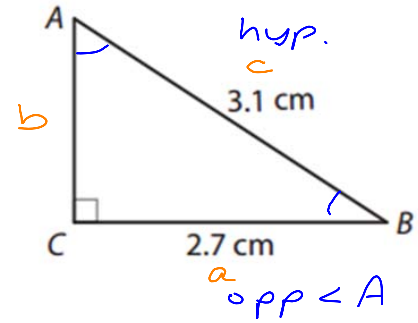
- **Given two side measures:**
  1. Use the **Pythagorean Theorem** to find the third side.
  2. Use **sine, cosine, or tangent** to find one acute angle measure.
  3. Use the **Triangle Sum Theorem** $(m\angle A + m\angle B + m\angle C = 180°)$ to find the second acute angle measure.

- **Given one side measure and one acute angle measure:**
  1. Use the **Triangle Sum Theorem** $(m\angle A + m\angle B + m\angle C = 180°)$ to find the second acute angle measure.
  2. Use sine, cosine, or tangent along with an acute angle and the known side measure to find another side measure.
  3. Use sine, cosine, or tangent along with an acute angle and the known side measure to find the remaining side measure.

Objective: Solve right triangles.

Ex) Solve the triangle. Round side lengths to the nearest tenth and angles to the nearest degree.

*Given two sides



① Pyth. Thm.: find b

$a^2 + b^2 = c^2$

$2.7^2 + b^2 = 3.1^2$

$b^2 = 3.1^2 - 2.7^2$

$b = \sqrt{3.1^2 - 2.7^2}$

$b \approx 1.5$ cm

② find ∠A

$\sin A = \frac{2.7}{3.1}$

$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{2.7}{3.1}\right)$

$A = \sin^{-1}\left(\frac{2.7}{3.1}\right)$

$m\angle A \approx 61^\circ$

③ find m∠B

Triangle Sum Thm.

$61^\circ + \angle B + 90^\circ = 180^\circ$

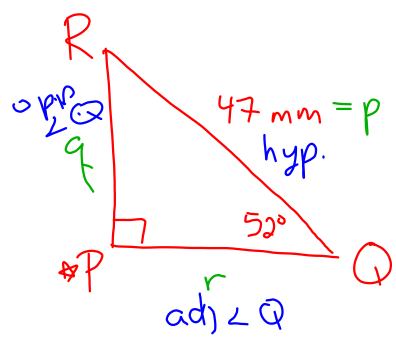
$m\angle B \approx 29^\circ$

Objective: Solve right triangles.

perpendicular to

Ex) Solve right ΔPQR with $\overline{PQ} \perp \overline{PR}$, $QR = 47\ mm$, and $m\angle Q = 52^\circ$. Round side lengths to the nearest tenth and angles to the nearest degree.

① draw and label the triangle



② find m∠R

Triangle Sum Theorem

$\angle R + \angle Q + \angle P = 180^\circ$

$\angle R + 52^\circ + 90^\circ = 180^\circ$

$m\angle R = 38^\circ$ — units of measure; exact measure

③ find side q.

$\sin Q = \frac{opp}{hyp}$

$47 \cdot \sin 52^\circ = \frac{q}{47} \cdot 47$

$q = 47 \cdot \sin 52^\circ$

$q \approx 37.0\ mm$ — approx. measure; units

④ find side r.

$\cos Q = \frac{adj}{hyp}$

$47 \cdot \cos 52^\circ = \frac{r}{47} \cdot 47$

$r = 47 \cdot \cos 52^\circ$

$r \approx 28.9\ mm$ — approx. measure; units

Objective: Solve right triangles.

Closure

When solving a right triangle, when should an inverse trigonometric function be used?

When solving a right triangle, an inverse trigonometric function should be used when two side measures are known in order to find one of the acute angle measures.