Objective: Verify trigonometric identities.

Concept

Trigonometric Identities allow us to rewrite trigonometric equations that model real-life situations so we can use our algebra skills to solve problems. The **key to verifying identities** and to solving trigonometric equations **is being able to use the fundamental identities to rewrite trigonometric expressions**.

Fundamental Trigonometric Identities

Pythagorean Identities	Quotient Identities	Reciprocal Identities	
$\sin^2\theta + \cos^2\theta = 1$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$1 + \cot^2\theta = \csc^2\theta$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$
$\tan^2\theta + 1 = \sec^2\theta$		$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Objective: Verify trigonometric identities.

Concept

A **Conditional Equation** is an equation that is **true for only some values of the domain of the variable**.

Examples are:

$x^2 - 6 = 10$; true for only $x = \pm 4$

$3x + 5 = 14$; true for only $x = 3$

An **Identity** is an equation that is **true for all values of the domain of the variable**.

Examples are:

$5x - 2 = 5x - 2$; true for all real numbers, x

$\sin^2\theta = 1 - \cos^2\theta$; true for all real numbers, θ

$\tan\theta = \frac{\sin\theta}{\cos\theta}$; true for all real numbers where $\tan\theta$ is defined.

Objective: Verify trigonometric identities.

Concept

There is no well-defined set of rules to follow in verifying trigonometric identities, the process is best learned through practice.

When you verify an identity you can't assume that the two sides of the equation are equal because you are trying to verify that they are equal. Therefore, **when verifying identities you cannot use algebraic procedures such as adding to both sides or multiplying the equation by a common denominator.**

Strategies for Verifying Trigonometric Identities

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.
4. When the preceding strategies don't help, try converting all terms to sines and cosines.
5. Always try *something*. Even making an attempt that leads to a dead end can provide insight.

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy: Work with the more complicated side.

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$$

$\dfrac{(\tan^2\theta + 1) - 1}{\sec^2\theta} = \sin^2\theta$ *pythagorean identity*

$\dfrac{\tan^2\theta}{\sec^2\theta} = \sin^2\theta$ *simplify*

$\tan^2\theta \cdot \dfrac{1}{\sec^2\theta} = \sin^2\theta$ *rewrite structure*

$\dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{1} = \sin^2\theta$ *quotient identity; reciprocal identity*

$\sin^2\theta = \sin^2\theta$ *simplify*

or

$\dfrac{\sec^2\theta}{\sec^2\theta} - \dfrac{1}{\sec^2\theta} = \sin^2\theta$ *separate terms*

$1 - \cos^2\theta = \sin^2\theta$ *simplify; reciprocal identity*

$\sin^2\theta = \sin^2\theta$ *pythagorean identity*

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy:
Combine fractions.

$$2\sec^2\alpha = \frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha}$$

$$2\sec^2\alpha = \frac{1}{1-\sin\alpha}\cdot\frac{1+\sin\alpha}{1+\sin\alpha} + \frac{1}{1+\sin\alpha}\cdot\frac{1-\sin\alpha}{1-\sin\alpha} \qquad \text{common denominator}$$

$$2\sec^2\alpha = \frac{1+\sin\alpha}{1-\sin^2\alpha} + \frac{1-\sin\alpha}{1-\sin^2\alpha} \qquad \textit{simplify}$$

$$2\sec^2\alpha = \frac{2}{1-\sin^2\alpha} \qquad \textit{simplify}$$

$$2\sec^2\alpha = \frac{2}{\cos^2\alpha} \qquad \textit{pythagorean identity}$$

$$2\sec^2\alpha = 2\cdot\frac{1}{\cos^2\alpha} \qquad \textit{rewrite structure}$$

$$2\sec^2\alpha = 2\sec^2\alpha \qquad \textit{reciprocal identity}$$

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy: Use Identities Before Multiplying.

$$(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$$

$(\sec^2 x)(\cos^2 x - 1) = -\tan^2 x$	*pythagorean identity*
$\sec^2 x \cdot -1(1 - \cos^2 x) = -\tan^2 x$	*rewrite / factor out* (-1)
$\sec^2 x \cdot -1(\sin^2 x) = -\tan^2 x$	*pythagorean identity*
$\frac{1}{\cos^2 x} \cdot -1(\sin^2 x) = -\tan^2 x$	*reciprocal identity*
$-1 \cdot \frac{\sin^2 x}{\cos^2 x} = -\tan^2 x$	*rewrite structure*
$-\tan^2 x = -\tan^2 x$	*quotient identity*

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy: Convert to Sines and Cosines.

$$\tan x \csc x = sec\, x$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \sec x$$ *quotient identity; reciprocal identity*

$$\frac{1}{\cos x} = \sec x$$ *simplify*

$$\sec x = \sec x$$ *reciprocal identity*

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy: Convert to Sines and Cosines.

$$\tan x + \cot x = \sec x \csc x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \csc x \quad \textit{quotient identity; quotient identity}$$

$$\frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} = \sec x \csc x \quad \text{common denominator}$$

$$\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} = \sec x \csc x \quad \textit{simplify}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \sec x \csc x \quad \textit{add fractions}$$

$$\frac{1}{\sin x \cos x} = \sec x \csc x \quad \textit{pythagorean identity}$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \csc x \quad \textit{rewrite structure}$$

$$\sec x \csc x = \sec x \csc x \quad \textit{reciprocal identity; reciprocal identity}$$

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy:
Multiply by a ratio of 1 using the conjugate.

$$\sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

$$\sec x + \tan x = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} \quad \text{multiply by ratio of 1}$$

$$\sec x + \tan x = \frac{\cos x + \cos x \sin x}{1-\sin^2 x} \quad \textit{simplify}$$

$$\sec x + \tan x = \frac{\cos x + \cos x \sin x}{\cos^2 x} \quad \textit{pythagorean identity}$$

$$\sec x + \tan x = \frac{\cos x}{\cos^2 x} + \frac{\cos x \sin x}{\cos^2 x} \quad \textit{rewrite structure}$$

$$\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} \quad \textit{simplify}$$

$$\sec x + \tan x = \sec x + \tan x \quad \textit{reciprocal identity; quotient identity}$$

Objective: Verify trigonometric identities.

Ex) Verify the identity.

Strategy:
Working with each side separately.

$$\frac{\cot^2\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$$

pythagorean identity $\dfrac{\csc^2\theta - 1}{1+\csc\theta} = \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$ rewrite structure

factor $\dfrac{(\csc\theta - 1)(\csc\theta + 1)}{1+\csc\theta} = \csc\theta - 1$ *reciprocal identity*; *simplify*

simplify(reduce) $\csc\theta - 1 = \csc\theta - 1$