Objective: Use Vertex Form to Graph and Write Quadratic Functions.

Concept

The parameters of a, h, and k create transformations on $f(x) = x^2$ that can be identified from the vertex form of a quadratic function.

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

If $a < 0$	the graph of the function will have an **x-axis reflection**
If $\|a\| > 1$	the graph of the function will have a **vertical stretch by a factor of $\|a\|$**
If $\|a\| < 1$	the graph of the function will have a **vertical compression by a factor of $\|a\|$**
If $h > 0$	the graph of the function is translated $\|h\|$ $units$ **right**
If $h < 0$	the graph of the function is translated $\|h\|$ $units$ **left**
If $k > 0$	the graph of the function is translated $\|k\|$ $units$ **up**
If $k < 0$	the graph of the function is translated $\|k\|$ $units$ **down**

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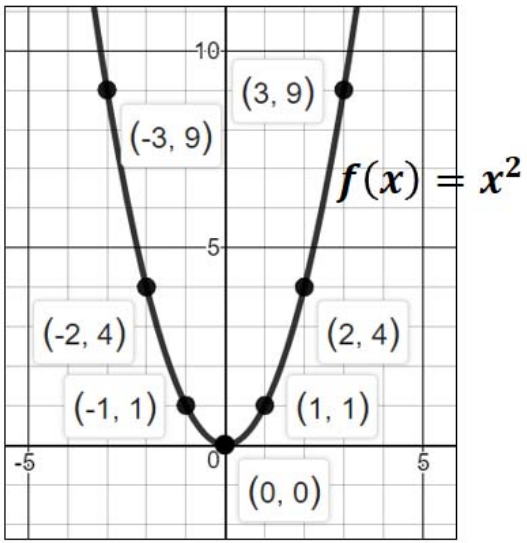
Concept

The **parent function of the family of quadratic functions** is $f(x) = x^2$. This is vertex form where $a = 1$, $h = 0$, and $k = 0$. The graph of a quadratic function is called a **parabola**.

x	$f(x) = x^2$
-3	$(-3)^2 = 9$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$
3	$(3)^2 = 9$



Objective: Use Vertex Form to Graph and Write Quadratic Functions.

a) Identify the transformations on $f(x) = x^2$, b) graph the quadratic function using the transformations.

Example) $g(x) = -2x^2 + 5$

$g(x) = -2(x - 0)^2 + 5$

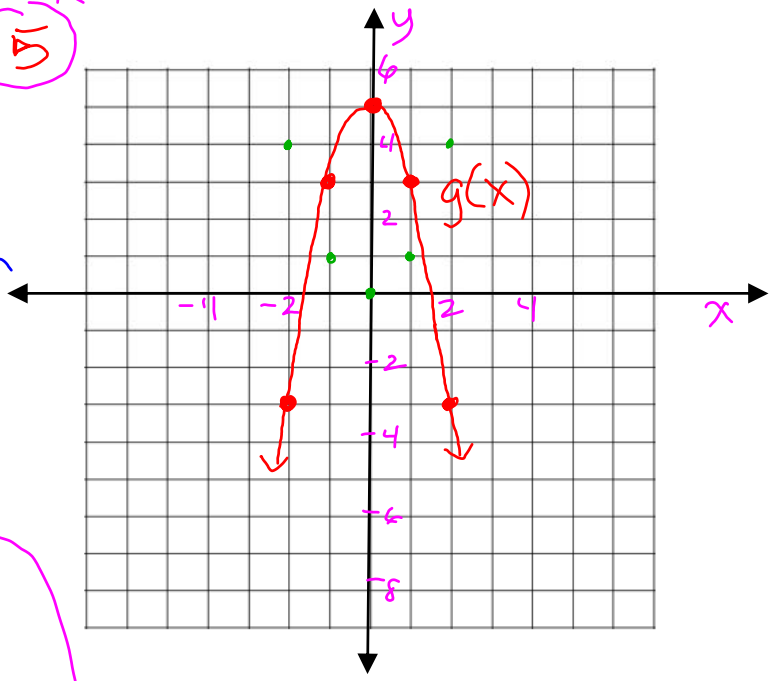
a opp = h k

$a = -2 < 0$ refl.

$|a| = |-2| = 2 > 1$ vert. stretch

$h = 0$ no horiz. trans.

$k = 5$ up



(a)
- x-axis reflection
- vertical stretch by a factor of 2
- translation up 5 units

Objective: Use Vertex Form to Graph and Write Quadratic Functions.

a) Identify the transformations on $f(x) = x^2$, b) graph the quadratic function using the transformations.

Example) $g(x) = \frac{1}{2}(x+3)^2$

$g(x) = \frac{1}{2}(x+3)^2 + 0$ (k)

a

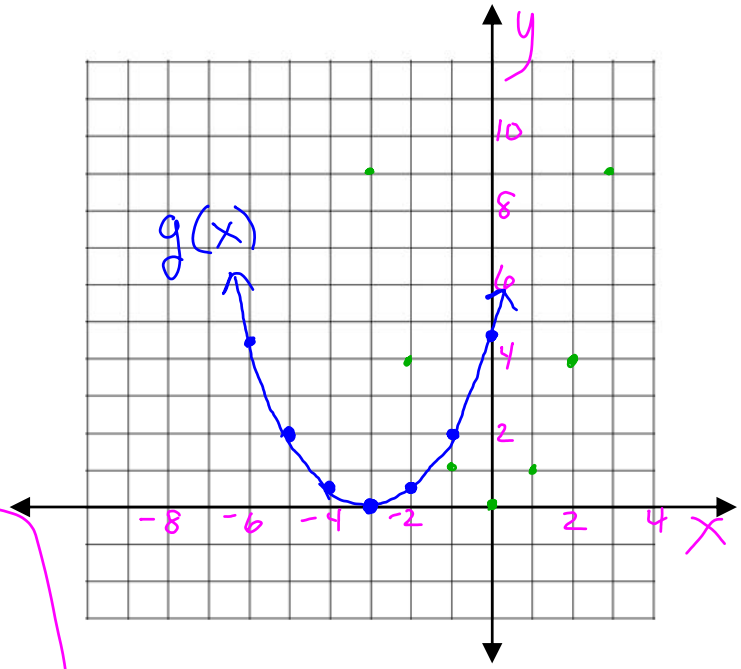
opp of h = −3

$a = \frac{1}{2} > 0$ no refl.

$|a| = |\frac{1}{2}| = \frac{1}{2} < 1$ vert. comp.

$h = -3$ left

$k = 0$ no vert. trans.



(a)
- vertical compression by a factor of $\frac{1}{2}$
- translation left 3 units

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Vertex Form of a Quadratic Function

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Objective: Use Vertex Form to Graph and Write Quadratic Functions.

a) Identify the transformations on $f(x) = x^2$, b) graph the quadratic function using the transformations.

Example) $g(x) = \frac{5}{2}(x-2)^2 - 3$ (a = $\frac{5}{2}$, opp. of h = −2, k)

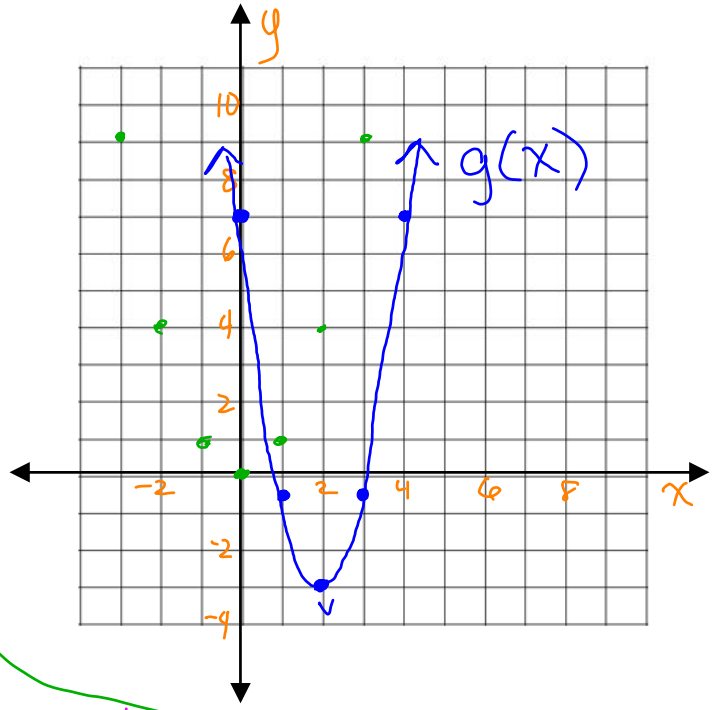
$a = \frac{5}{2} > 0$ no refl.

$|a| = \left|\frac{5}{2}\right| = \frac{5}{2} > 1$ vert. stretch

$2\frac{1}{2}$

$h = 2$ right

$k = -3$ down

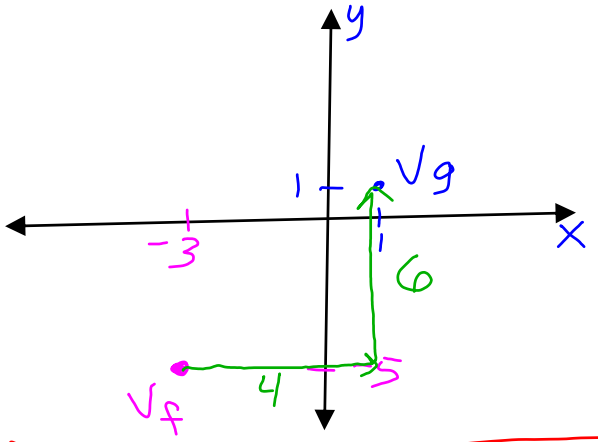


(a)
- vertical stretch by a factor of 5/2
- translation right 2 units and down 3 units

Objective: Use Vertex Form to Graph and Write Quadratic Functions.

Ex) Given the graph of $f(x) = (x+3)^2 - 5$ (parent/start) how does Mary create the graph of $g(x) = (x-1)^2 + 1$? (end here)

Strategy: graph the vertex of $f(x)$ and the vertex of $g(x)$, then determine the translation(s) from $f(x)$ to $g(x)$. The vertex of a parabola is at (h, k).



① vertex of f(x) is $(-3, -5)$

$h = -3$ $k = -5$

② vertex of g(x) is $(1, 1)$

$h = 1$ $k = 1$

Mary can create the graph of g(x) by translating f(x) right 4 units and up 6 units.

Objective: Use Vertex Form to Graph and Write Quadratic Functions.

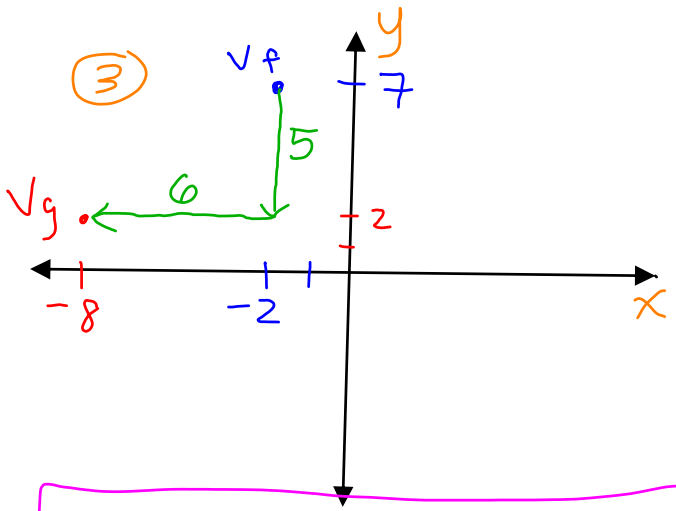
Ex) Given the graph of $f(x) = (x+2)^2 + 7$ how does Mary create the graph of $g(x) = (x+8)^2 + 2$?

① vertex of f(x) is (-2, 7)

h = -2 k = 7

② vertex of g(x) is (-8, 2)

h = -8 k = 2



Mary can create g(x) by translating f(x) down 5 units and left 6 units.

Objective: Use Vertex Form to Graph and Write Quadratic Functions.

Ex) Write a quadratic function in the form $f(x) = a(x-h)^2 + k$ that has the given transformations.

- a reflection over the x-axis → a is negative
- a vertical stretch by a factor of 3 → $a = -3$
- a translation left 4 units and down 7 units

$h = -4$ $k = -7$

$$f(x) = -3(x-(-4))^2 + -7$$

$$f(x) = -3(x+4)^2 - 7$$

Objective: Use Vertex Form to Graph and Write Quadratic Functions.

Ex) Write a quadratic function in the form $f(x) = a(x-h)^2 + k$ that has the given transformations.

- a vertical compression by a factor of $\frac{3}{7}$ → $a = \frac{3}{7}$
- a translation right 12 units

$h = 12 \quad k = 0$

$$f(x) = \frac{3}{7}(x-12)^2 + 0$$

$$\boxed{f(x) = \frac{3}{7}(x-12)^2}$$

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Closure

Nina is trying to write an equation for the function represented by the graph of a parabola that is a transformation of $f(x) = (x-3)^2-1$. The graph has been translated 4 units to the right and 2 units up. Her function is shown below. Explain Nina's error and write the correct function.

$$g(x) = (x-7)^2 + 2$$

Nina should have added 2 to the -1, giving a k value of 1. The correct function is $g(x) = (x-7)^2 + 1$.