Objective: Multiply and Divide Rational Expressions.

Concept

To multiply rational expressions, we use the same basic procedure as when multiplying numeric fractions.

To Multiply Two or More Rational Expressions:

1. **Factor** all numerators and denominators. Including GCFs.
2. **Reduce all common factors** in the numerator and denominator.
3. **Multiply straight across**, removing all parentheses. Write the numerator and denominator in standard form.

Objective: Multiply and Divide Rational Expressions.

Ex) Find the product. State any excluded values.

$$\frac{3x^2}{x^2-2x-8}\cdot\frac{2x^2-6x-20}{x^2-2x-15}$$

$(x-4)(x+2)$ $(x-5)(x+3)$

(a) excluded values

$x-4=0,\quad x+2=0,\quad x-5=0,\quad x+3=0$

$$x \neq -3, -2, 4, 5$$

(b)

$$\frac{3\cdot x^2}{(x-4)(x+2)}\cdot\frac{2(x^2-3x-10)}{(x-5)(x+3)}$$

$$\rightarrow \frac{3x^2\cdot 2}{(x-4)(x+3)} \rightarrow \frac{6x^2}{x^2-x-12}$$

Objective: Multiply and Divide Rational Expressions.

Ex) Find the product. State any excluded values.

$$\frac{x^2+8x}{3x^2+15x}\cdot\frac{6x+30}{x^2+10x+16}$$

$3x(x+5)$ $(x+8)(x+2)$

(a) excluded values

$\frac{3x}{3}=\frac{0}{3}$, $x+5=0$, $x+8=0$, $x+2=0$

$x\neq 0$ $x\neq -5$ $x\neq -8$ $x\neq -2$

$$x\neq -8,-5,-2,0$$

(b) product

$$\frac{x(x+8)}{3x(x+5)}\cdot\frac{6(x+5)}{(x+8)(x+2)}$$

$$\rightarrow \frac{2}{x+2}$$

Objective: Multiply and Divide Rational Expressions.

Concept

Properties of Rational Exponents		
For all nonzero real numbers a and b and rational numbers m and n		
Words	**Numbers**	**Algebra**
Product of Powers Property: to multiply powers with the same base, add the exponents	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property: to divide powers with the same base, subtract the exponents	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$ or $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
Negative Exponent Property: moving a power from numerator to denominator or vice versa changes the sign on the exponent	$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{6}$ $\frac{1}{36^{-\frac{1}{2}}} = \frac{36^{\frac{1}{2}}}{1} = \frac{6}{1} = 6$	$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$
Zero Exponent Property: any monomial to a power of 0 is equal to 1	$(3)^0 = 1$	$(a)^0 = 1$

Objective: Multiply and Divide Rational Expressions.

Ex) Find the product. Assume all denominators are not equal to zero.

$$\frac{(-3x^{-5}y^{-3})}{(-2x^{2}y^{-4})} \cdot \frac{(4x^{-6}y)}{(9xy^{-2})}$$

① neg. exp. rule

$$\frac{-3 \cdot y^{4}}{-2 \cdot x^{2} \cdot x^{5} \cdot y^{3}} \cdot \frac{4 \cdot y \cdot y^{2}}{9 \cdot x \cdot x^{6}}$$

②

$$\frac{+3 \cdot 4 \cdot y^{4} \cdot y \cdot y^{2}}{+2 \cdot 9 \cdot x^{2} \cdot x^{5} \cdot x \cdot x^{6} \cdot y^{3}}$$

③

$$\frac{2y^{7}}{3x^{14}y^{3}} \rightarrow \frac{2y^{7-3}}{3x^{14}} \rightarrow \boxed{\frac{2y^{4}}{3x^{14}}}$$

Objective: Multiply and Divide Rational Expressions.

Concept

To divide rational expressions, we use the same basic procedure as when dividing numeric fractions.

To Divide Two Rational Expressions:

1. Rewrite the problem as **Multiplication by the Reciprocal**.
2. **Factor** all numerators and denominators. Including GCFs.
3. **Reduce all common factors** in the numerator and denominator. Don't forget to reduce common factors in the GCFs.
4. **Multiply straight across**, removing all parentheses. Write the numerator and denominator in standard form.

Objective: Multiply and Divide Rational Expressions.

Ex) Find the quotient. State any excluded values.

$$\frac{6x^2+12x}{3x-30} \div \frac{2x^2+3x-2}{x^2-10x}$$

$3(x-10)$ $\quad x(x-10)$ $\quad (2x-1)(x+2)$

(a) excluded values (use both denominators and the second numerator)

$3 \neq 0$ $\quad x-10=0$ $\quad x=0$ $\quad 2x-1=0$ $\quad x+2=0$

$x \neq 10$ $\quad x \neq 0$ $\quad x \neq \frac{1}{2}$ $\quad x \neq -2$

$$x \neq -2, 0, \tfrac{1}{2}, 10$$

(b) (1) Rewrite as multiply by the reciprocal.

$$\frac{6x^2+12x}{3(x-10)} \cdot \frac{x(x-10)}{(2x-1)(x+2)}$$

(2)

$$\frac{6x(x+2)}{3(x-10)} \cdot \frac{x(x-10)}{(2x-1)(x+2)}$$

$$\frac{2 \cdot x \cdot x}{2x-1} \rightarrow \boxed{\frac{2x^2}{2x-1}}$$

Objective: Multiply and Divide Rational Expressions.

Ex) Find the quotient. State any excluded values. (b) (a)

$$\frac{x^2+14x+49}{x^2} \div \frac{x^2+9x+14}{x^2+2x}$$

$x \cdot x$ $\quad$ $(x+7)(x+2)$ $\quad$ $x(x+2)$

(a) excluded values (use both denominators and the second numerator)

$x = 0$ $\quad$ $x+2=0$ $\quad$ $x+7=0$

$x \neq 0$ $\quad$ $x \neq -2$ $\quad$ $x \neq -7$

$x \neq -7, -2, 0$

(b) (1) Rewrite as multiply by the reciprocal

$$\frac{(x+7)(x+7)}{x} \cdot \frac{x(x+2)}{(x+7)(x+2)} \rightarrow \frac{x+7}{x}$$

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Ex) Find the quotient. Assume all denominators are not equal to zero.

$$\frac{(-3x^{-5}y^{-3})}{(-2x^{2}y^{-4})} \div \frac{(4x^{-6}y)}{(9xy^{-2})}$$

① Rewrite as multiply by the reciprocal

$$\frac{-3x^{-5}y^{-3}}{-2x^{2}y^{-4}} \cdot \frac{9xy^{-2}}{4x^{-6}y}$$

② neg. exponent rule

$$\frac{-3 \cdot y^{4}}{-2 \cdot x^{2} \cdot x^{5} \cdot y^{3}} \cdot \frac{9 \cdot x \cdot x^{6}}{4 \cdot y \cdot y^{2}}$$

$$\frac{+3 \cdot 9 \cdot x^{1+6} \cdot y^{4}}{+2 \cdot 4 \cdot x^{2+5} \cdot y^{3+1+2}}$$

$$\frac{27x^{7}y^{4}}{8x^{7}y^{6}} \rightarrow \frac{27x^{7-7}}{8y^{6-4}} \quad (x^{0}=1)$$

$$\rightarrow \boxed{\frac{27}{8y^{2}}}$$

Objective: Multiply and Divide Rational Expressions.

Closure

A set of numbers or expressions is said to be closed, or to have **closure**, under a given operation if the result of the operation on any two numbers or expressions in the set is also in the set.

The **Set of Rational Expressions** includes all **expressions of the form** $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

Is the Set of Rational Expressions closed under multiplication?

Yes, because the product of two rational expressions will always be another rational expression.

$$\frac{p(x)}{q(x)} \cdot \frac{r(x)}{s(x)} = \frac{p(x)r(x)}{q(x)s(x)}$$

Is the Set of Rational Expressions closed under division?

Yes, because the quotient of two rational expressions will always be another rational expression.

$$\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)s(x)}{q(x)r(x)}$$