Objective: Graph cubic functions using transformations

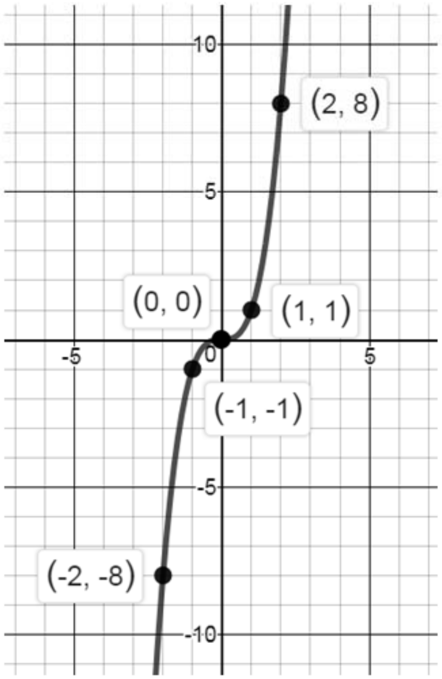
Concept

The parent function of the family of cubic functions is $f(x) = x^3$.
A cubic function in the form $f(x) = a(x-h)^3 + k$ can be graphed using transformations using the key points of the parent function.

For $f(x) = x^3$, the point $(0,0)$ is called the point of inflection. This point is only affected by translations. All other points are affected by all types of transformations.

x	$f(x) = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$

$f(x) = x^3$



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Concept

Recall: **The order in which transformations should be performed follow the Order of Operations.** Transformations that involve multiplication should be done first (reflections, stretches, compressions). Transformations that involve addition should be done second (translations right/left/up/down). There are exceptions and variations to this procedure, but this procedure always works.

One Procedure for Graphing a Cubic Function Using Transformations

1. Determine the transformations.
2. Translate $(0,0)$ to determine the new point of inflection.
3. Perform any reflection, stretch, and/or compression on the other key points of the parent function and then translate these points.
4. Draw in a smooth curve.

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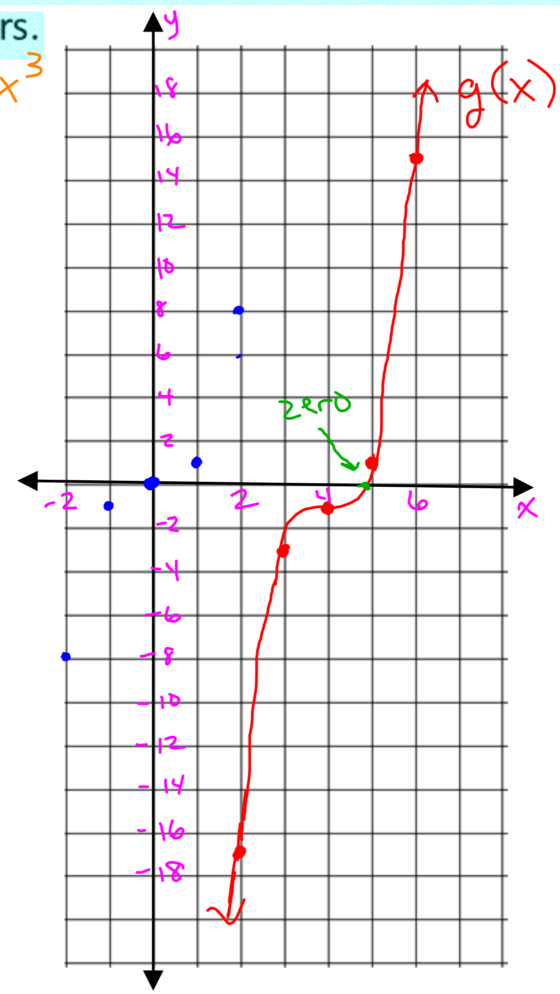
Ex) A) Graph using transformations. B) Determine the zero or the interval of consecutive integers in which the zero occurs.

$g(x) = 2(x-4)^3 - 1$ → parent $f(x) = x^3$

$a = 2$ vert. stretch

$h = 4$ right

$k = -1$ down



(B) The zero of g(x) is in the interval (4, 5).

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Ex) A) Graph using transformations. B) Determine the zero or the interval of consecutive integers in which the zero occurs.

$h(x) = -x^3 - 1$

-1 → a; -1 → k

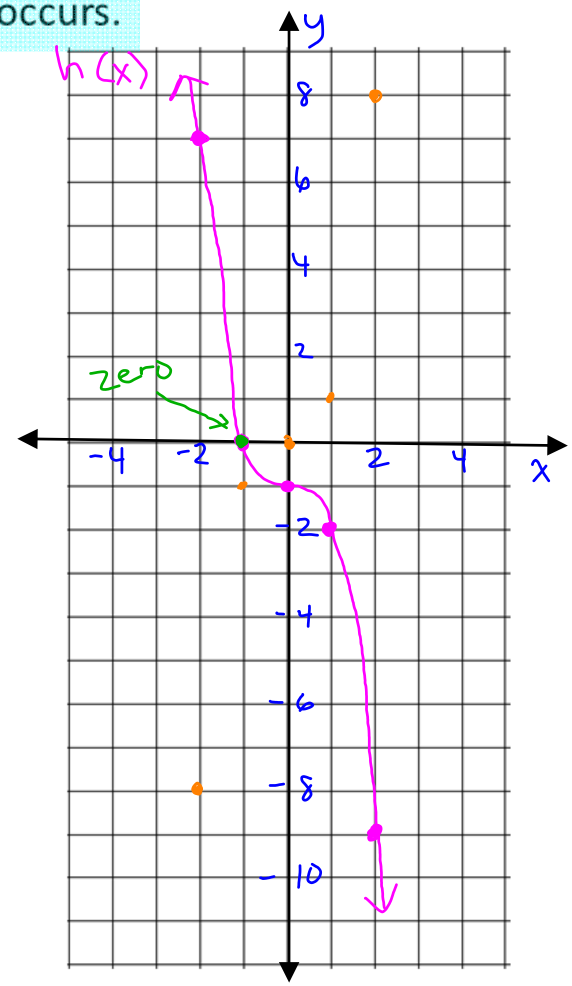
parent $f(x) = x^3$

$a = -1$ x-axis refl.

$|a| = |-1| = 1 = 1$ no stretch, no comp.

$k = -1$ down 1

B) The zero of $h(x)$ is -1.



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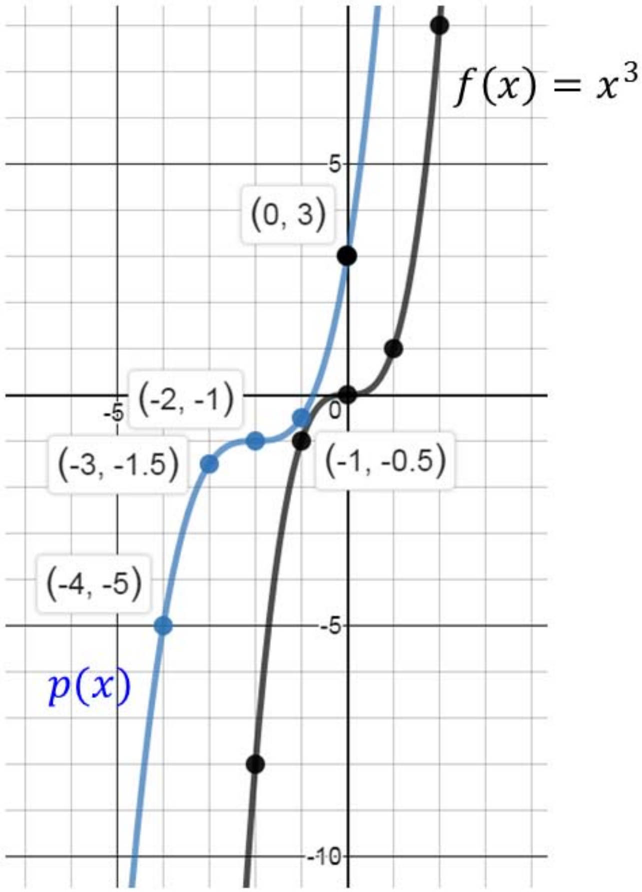
Practice) A) Graph using transformations. B) Determine the zero or the interval of consecutive integers in which the zero occurs.

$$p(x) = \frac{1}{2}(x+2)^3 - 1$$

B) The zero is in the interval $(-1,0)$.



Objective: Graph cubic functions using transformations

Concept

Given a parent function $f(x)$, $g(x) = af(x-h) + k$ and $g(x) = f\left(\frac{1}{b}(x-h)\right) + k$ can be graphed by identifying the transformations and then transforming the key points of $f(x)$.

Objective: Graph cubic functions using transformations

Ex) Given the parent function $f(x) = x^3$, graph $g(x)$ using transformations.

$g(x) = f(2x) + 5$ (k)   ($\frac{1}{b}$)

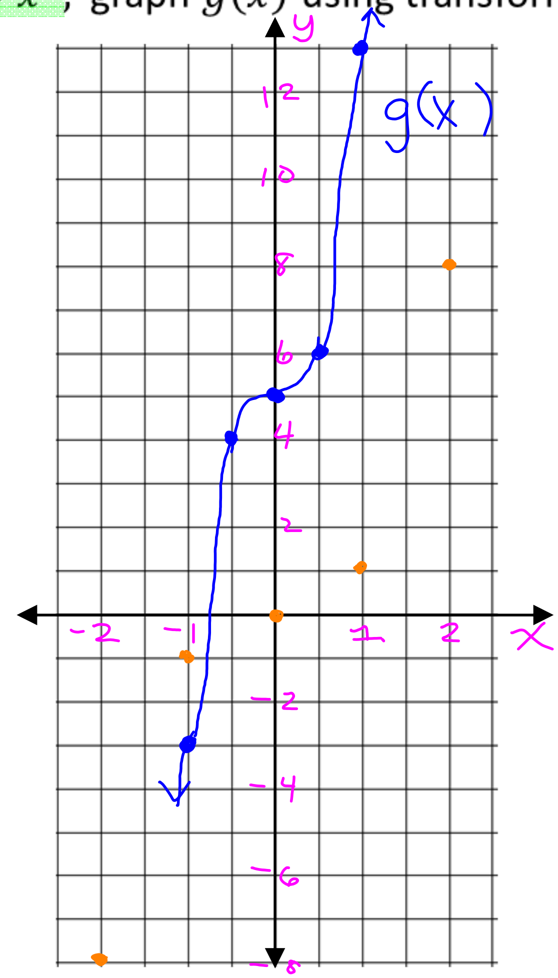
$\frac{1}{b} = 2 = \frac{2}{1}$

$b = \frac{1}{2}$ no refl.

$|b| = \left|\frac{1}{2}\right| = \frac{1}{2} \leq 1$

horiz. comp.

$k = 5$ up 5



Objective: Graph cubic functions using transformations

Ex) Given the parent function $f(x) = x^3$, graph $g(x)$ using transformations.

$g(x) = f(-(x-3))$

$\frac{1}{b}$

$\frac{1}{b} = -1 = \frac{-1}{1}$

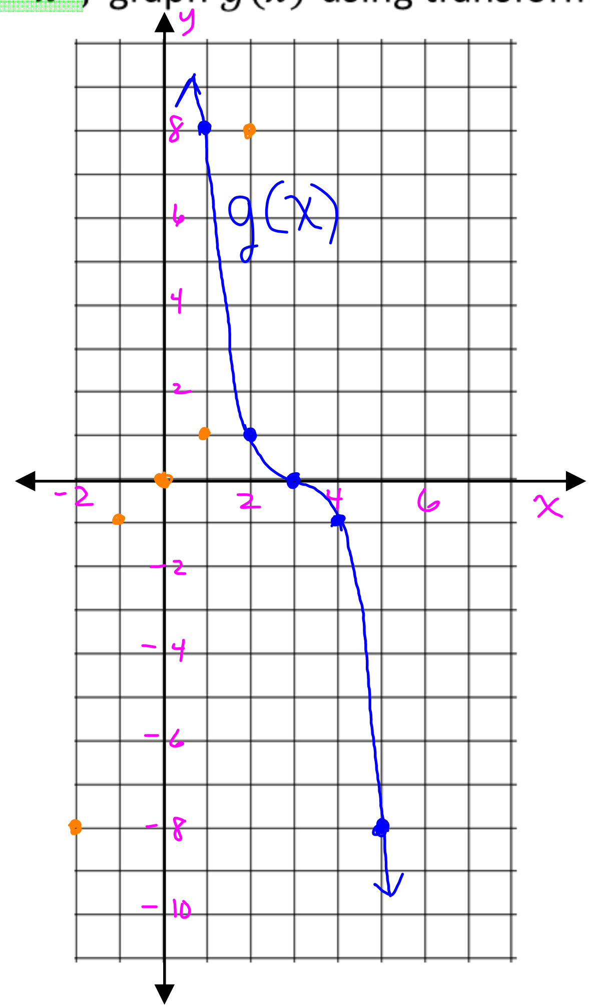
$b = -1$ y-axis refl.

$|b| = |-1| = 1 = 1$

no stretch

no comp.

$h = 3$ right 3

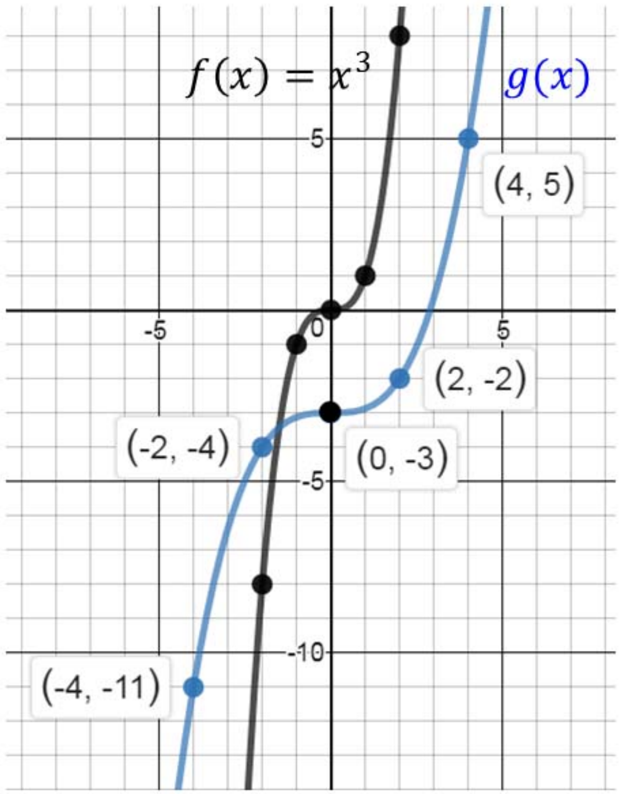


Objective: Graph cubic functions using transformations

Practice) Given the parent function $f(x) = x^3$, graph $g(x)$ using transformations.

$$g(x) = f\left(\frac{1}{2}x\right) - 3$$

Horizontal stretch,
factor of 2
Down 3 units



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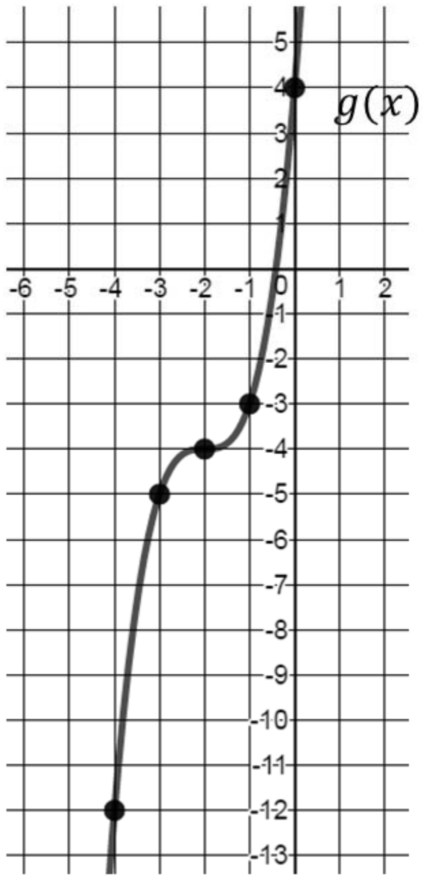
Closure

When compared to $f(x) = x^3$, which function represents the graph of $g(x)$?



1. $g(x) = (x-2)^3 + 4$
2. $g(x) = (x+2)^3 + 4$
3. $g(x) = (x-2)^3 - 4$
4. $g(x) = (x+2)^3 - 4$
5. $g(x) = (x-4)^3 - 2$
6. $g(x) = (x+4)^3 - 2$
7. $g(x) = (x-4)^3 + 2$
8. $g(x) = (x+4)^3 + 2$