Objective: Graph polynomial functions from factored form

Concept

Polynomial Functions and End Behavior

The end behavior of a polynomial function is determined by two characteristics.

1. The **degree** (the highest power of the independent variable) of the function: **even or odd**
2. The **sign of the leading coefficient** (the constant factor, a, of the first term when the function is written in standard form): **positive or negative**

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Concept

Odd Degree (x, x^3, x^5) Polynomial Functions

Positive Leading Coefficient

End Behavior

$as\ x \to -\infty, f(x) \to -\infty$

$as\ x \to +\infty, f(x) \to +\infty$

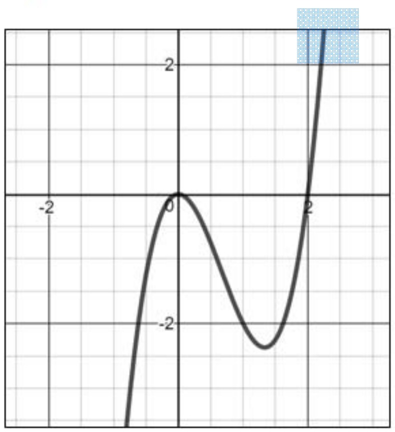
Negative Leading Coefficient

End Behavior

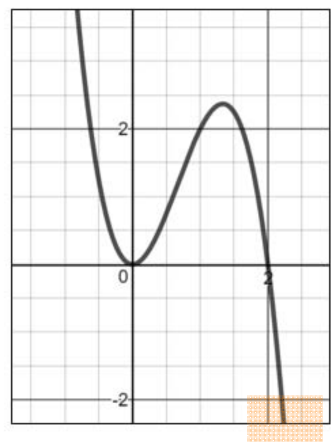
$as\ x \to -\infty, f(x) \to +\infty$

$as\ x \to +\infty, f(x) \to -\infty$

$$f(x) = 2x^3 - 4x^2$$



$$f(x) = -2x^3 + 4x^2$$



Objective: Graph polynomial functions from factored form

Concept

Even Degree (x^2, x^4, x^6) Polynomial Functions

Positive Leading Coefficient

End Behavior

$as\ x \to -\infty, f(x) \to +\infty$

$as\ x \to +\infty, f(x) \to +\infty$

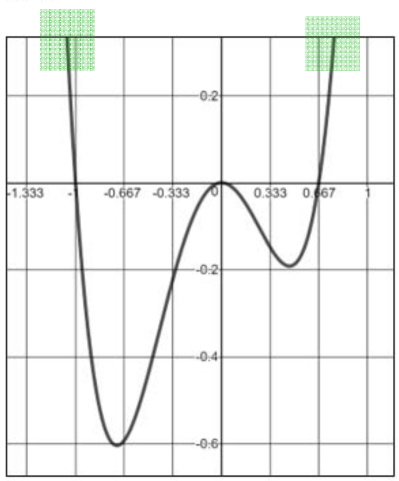
Negative Leading Coefficient

End Behavior

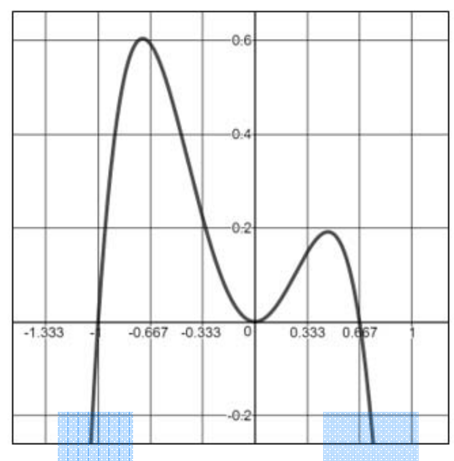
$as\ x \to -\infty, f(x) \to -\infty$

$as\ x \to +\infty, f(x) \to -\infty$

$$f(x) = 3x^4 + x^3 - 2x^2$$



$$f(x) = -3x^4 - x^3 + 2x^2$$



Objective: Graph polynomial functions from factored form

Concept

Steps to Graph a Polynomial Function from Factored Form

ex) $x, 1, 2x, x^2$

1. Find the first term. Multiply any monomial factor and all variable terms from all other factors. $(3x+4)$
2. Determine the end behavior using the first term.
3. Find the zeros of the function. (Include multiplicity)
4. Sketch a smooth curve.

Objective: Graph polynomial functions from factored form

Ex) For each polynomial function: a) state the end behavior, b) state the exact values of the real zeros (include multiplicity), c) sketch the graph.

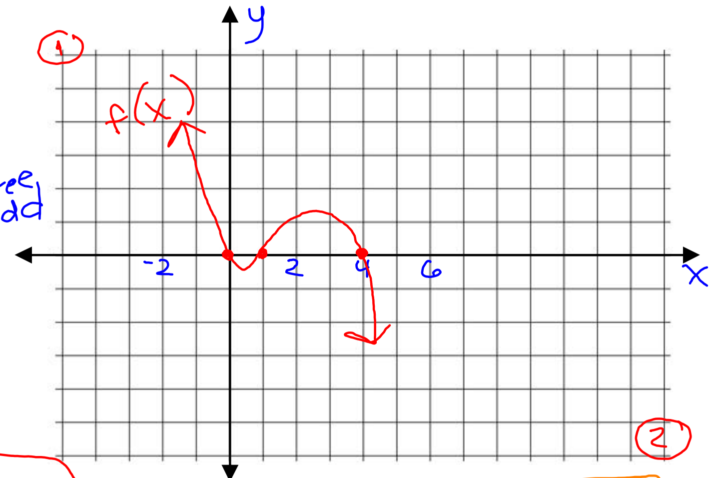
$f(x) = x(x-4)(1-x)$

① find the first term

$x \cdot x \cdot -1x = -1x$

③ degree is odd

* negative



ⓐ end behavior

as $x \to -\infty$, $f(x) \to +\infty$

as $x \to +\infty$, $f(x) \to -\infty$

ⓑ zeros = 0, 1, 4

(no multiplicity)

③ find the zeros

$0 = x(x-4)(1-x)$

$x = 0$, $x - 4 = 0$, $1 - x = 0$

$+4 \quad +4$ $\quad$ $+x \quad +x$

$x = 4$ $\quad$ $1 = x$

Objective: Graph polynomial functions from factored form

Ex) For each polynomial function: a) state the end behavior, b) state the values of the real zeros (include multiplicity), c) sketch the graph.

$f(x) = (x+5)^2(3x-2)(2x+3)$

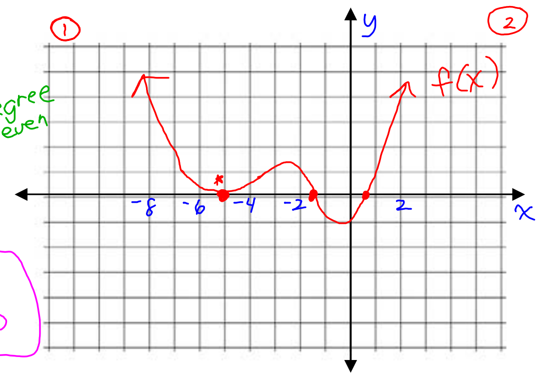
or $f(x) = 1(x+5)(x+5)(3x-2)(2x+3)$

① find the first term

$1 \cdot x^2 \cdot 3x \cdot 2x = 6x^4$

or

$1 \cdot x \cdot x \cdot 3x \cdot 2x = 6x^4$ degree is even, positive



ⓐ end behavior

$\boxed{\begin{array}{l} \text{as } x \to -\infty, f(x) \to +\infty \\ \text{as } x \to +\infty, f(x) \to +\infty \end{array}}$

③ find the zeros

$0 = (x+5)(x+5)(3x-2)(2x+3)$

$x+5=0$, $-5 \quad -5$, $x = -5$

$x+5=0$, $-5 \quad -5$, $x = -5$

$3x-2=0$, $+2 \quad +2$, $\frac{3x}{3} = \frac{2}{3}$, $x = \frac{2}{3}$

$2x+3=0$, $-3 \quad -3$, $\frac{2x}{2} = \frac{-3}{2}$, $x = \frac{-3}{2} = -1\frac{1}{2}$

$2\overline{)3}$ → $1\frac{1}{2}$

ⓑ $\boxed{\text{zeros} = -5, -5, -1\tfrac{1}{2}, \tfrac{2}{3}}$

or

$\boxed{-5 \text{ (multiplicity } \times 2), -1\tfrac{1}{2}, \tfrac{2}{3}}$

Objective: Graph polynomial functions from factored form

Ex) For each polynomial function: a) state the end behavior, b) state the values of the real zeros (include multiplicity), c) sketch the graph.

$f(x) = (x^2 - 20)(9 - x^2)$

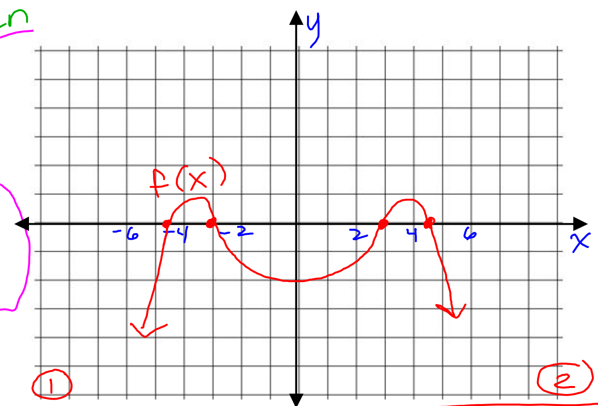
① find the first term

$1 \cdot x^2 \cdot -1x^2 = -1x^4$ ④ degree is even

* negative

ⓐ end behavior

$\boxed{\begin{array}{l} \text{as } x \to -\infty,\ f(x) \to -\infty \\ \text{as } x \to +\infty,\ f(x) \to -\infty \end{array}}$



③ find the zeros

$0 = (x^2 - 20)(9 - x^2)$

$0 = (x^2 - 20)(3 - x)(3 + x)$

$x^2 - 20 = 0$ $\quad 3 - x = 0$ $\quad 3 + x = 0$

$+20 \quad +20$ $\quad +x \quad +x$ $\quad -3 \quad -3$

ⓑ $\boxed{\text{zeros} = -2\sqrt{5}, -3, 3, 2\sqrt{5}}$

(no multiplicity)

$x^2 = 20$ $\quad 3 = x$ $\quad x = -3$

$\sqrt{x^2} = \pm\sqrt{20}$

$\sqrt{4}\sqrt{5}$

$x = \pm 2\sqrt{5}$

$\sqrt{16} \quad \sqrt{20} \quad \sqrt{25}$

$4 \quad \approx 4.5 \quad 5$