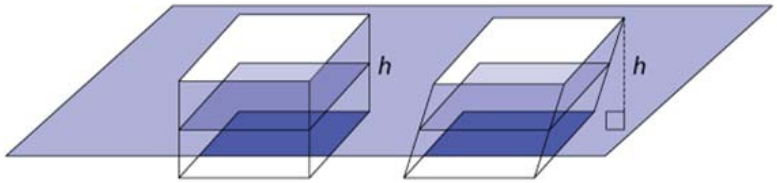
Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

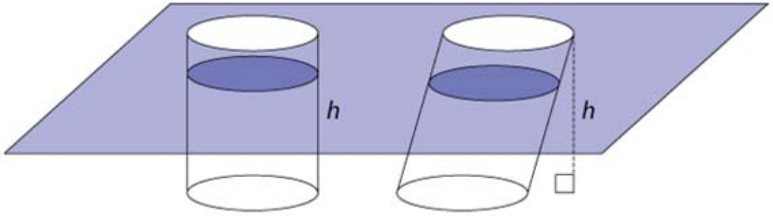
Cavalieri's Principle

Cavalieri's Principle says that if two solids have the same height and the same cross-sectional area at every level, then the two solids have the same volume. This means the **volume of an oblique prism or oblique cylinder** can be found using the same volume formula for right prisms and right cylinders, $\boldsymbol{V = B \cdot h}$, where B is the area of the base and h is the height.

right prism oblique prism

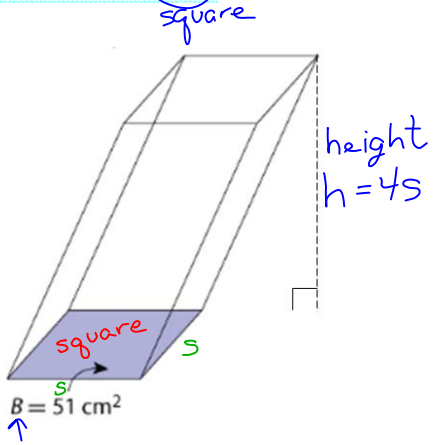


right cylinder oblique cylinder



Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Ex) The height of the oblique square-based prism is four times that of the side length of the base. What is the volume of the prism? Round to the nearest tenth.



Base Area

① $V = B \cdot h$

$V = s^2 \cdot h$

② find s, side length

$B = s^2$

$51\text{ cm}^2 = s^2$

$\sqrt{51\text{ cm}^2} = \sqrt{s^2}$

$s \approx 7.14\text{ cm}$

③ find height, h

$h = 4s$

$h = 4(7.14\text{ cm})$

$h = 28.56\text{ cm}$

④ volume

$V = B \cdot h$

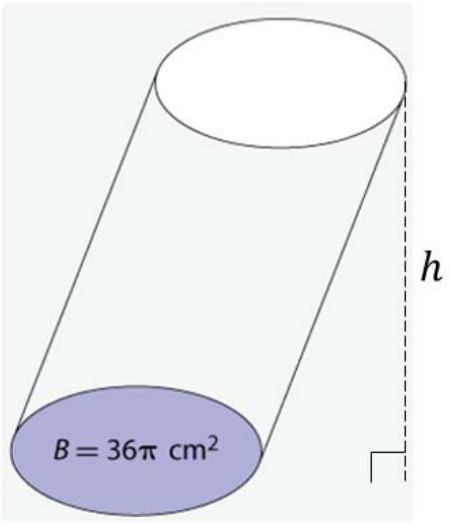
$V = (51\text{cm}^2)(28.56\text{ cm})$

$V \approx 1456.6\text{ cm}^3$

⑤ The volume of the prism is about 1456.6 cm^3.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Practice) The height of this oblique cylinder is three times that of its radius. What is the volume of the cylinder in cubic centimeters. Round to the nearest tenth.



1. Find the radius.

$$\pi \cdot r^2 = 36\pi\ cm^2$$

$$r^2 = 36\ cm^2$$

$$r = 6\ cm$$

2. Find the height.

$$h = 3 \cdot 6\ cm = 18\ cm$$

3. Calculate Volume

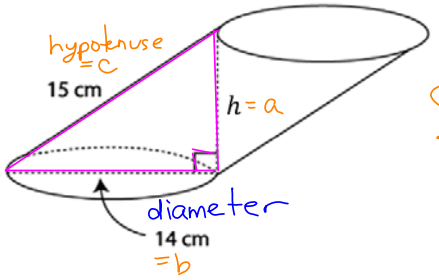
$$V = B \cdot h = 36\pi\ cm^2 \cdot 18\ cm \approx 2035.8\ cm^3$$

The volume of the cylinder is about $2035.8\ cm^3$.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Ex) A vase in the shape of an oblique cylinder has the dimensions shown. What is the volume of the vase in liters? (Hint: Use the right triangle in the cylinder to find its height.) Round to the nearest tenth.

Use the conversion $1\ cm^3 = 0.001\ liters$



① $V = B \cdot h$

$V = \pi r^2 \cdot h$

② find height, h

★ use the Pythagorean Theorem

$a^2 + b^2 = c^2$

$h^2 + 14^2 = 15^2$

$h^2 + 196 = 225$

$-196 \quad -196$

$h^2 = 29$

$\sqrt{h^2} = \sqrt{29}$

$h \approx 5.4\ cm$

③ find radius, r

$r = \frac{\text{diameter}}{2} = \frac{14cm}{2} = 7cm$

④ volume $V = B \cdot h$

$V = \pi r^2 \cdot h$

$V = \pi (7cm)^2 \cdot 5.4cm$

$= \pi \cdot 49cm^2 \cdot 5.4cm$

$V \approx 831.3\ cm^3$

⑤ convert cm^3 to liters

$\frac{831.3\ cm^3}{1} \cdot \frac{0.001\ \text{liter}}{1\ cm^3}$

≈ 0.8 liters

⑥ The volume of the vase is about 0.8 liters.

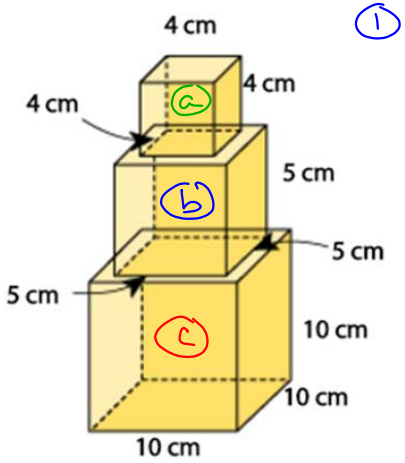
Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Concept

A **composite solid** is a solid made up of more than one solid. The solids can be the same kind (all prisms) or different kinds (a prism and cylinder).

To find the **volume of a composite solid**, find the volumes of the individual solids and then add the volumes together.

Ex) Find the volume of the composite solid in cubic centimeters.



① Three cubes (rectangular prisms)

$V = B \cdot h$

$V = s^2 \cdot s \rightarrow V = s^3$

② find the volume of each cube

ⓐ $V = (4\text{ cm})^3 = 64\text{ cm}^3$

ⓑ $V = (5\text{ cm})^3 = 125\text{ cm}^3$

ⓒ $V = (10\text{ cm})^3 = 1000\text{ cm}^3$

③ add the volumes

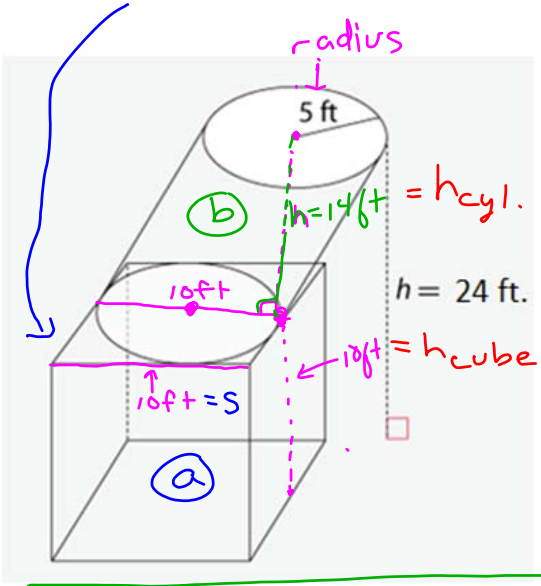
Total volume $= 64\text{cm}^3 + 125\text{ cm}^3 + 1000\text{ cm}^3$

$= 1189\text{ cm}^3$

④ The volume of the composite solid is 1189 cm^3.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Ex) Find the volume of the composite solid, which is made up of an oblique cylinder and a cube, in cubic feet. Round to the nearest tenth.



① find the height of each figure

② volume of each figure

ⓐ $V = s^3$

$V = (10\text{ ft})^3 = 1000\text{ ft}^3$

ⓑ $V = B \cdot h$

$V = \pi r^2 \cdot h$

$V = \pi (5\text{ ft})^2 \cdot 14\text{ ft}$

$= \pi \cdot 25\text{ ft}^2 \cdot 14\text{ ft}$

$V \approx 1099.6\text{ ft}^3$

③ add the volumes

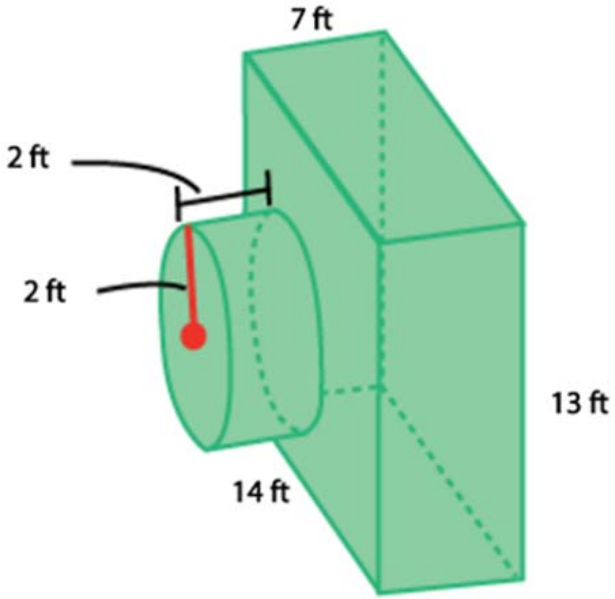
total volume $= 1000\text{ ft}^3 + 1099.6\text{ ft}^3$

$\approx 2099.6\text{ ft}^3$

④ The volume of the composite solid is about 2099.6 ft^3.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Practice) Find the volume of the composite figure in cubic feet. Round to the nearest tenth.



1. Find the volume of the prism.

$$V = 7\,ft \cdot 14\,ft \cdot 13\,ft = 1274\,ft^3$$

2. Find the volume of the cylinder.

$$V = \pi \cdot (2\,ft)^2 \cdot 2\,ft \approx 25.1\,ft^3$$

3. Add the volumes.

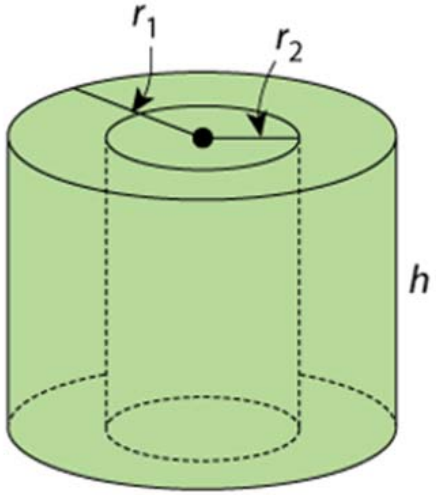
$$1274\,ft^3 + 25.1\,ft^3 \approx 1299.1\,ft^3$$

The volume of the composite solid is about $1299.1\,ft^3$.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Concept

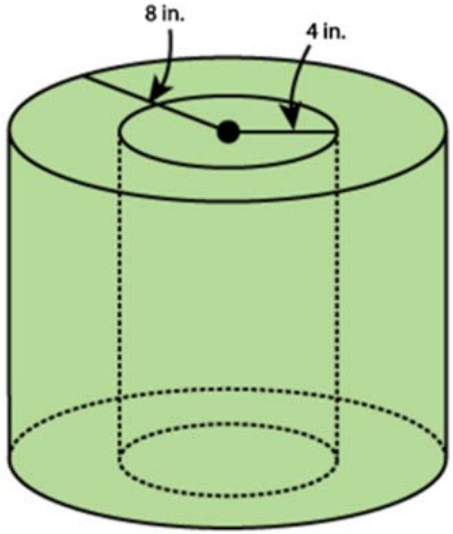
The pipe consists of two concentric cylinders, with the inner cylinder hollowed out. Describe how you could calculate the volume of the solid pipe.



Subtract the volume of the smaller (hollow) cylinder from the larger cylinder.

Objective: Find the volume of composite solids, including those with oblique prisms and cylinders.

Practice) The pipe shown consists of two concentric cylinders, with the inner cylinder hollowed out. Find the volume of the solid portion of pipe. Round to the nearest tenth.



1. Find the volume of the larger cylinder.

$$V = \pi \cdot (8\,in)^2 \cdot 17\,in = 3418.1\,in^3$$

2. Find the volume of the smaller cylinder.

$$V = \pi \cdot (4\,in)^2 \cdot 17\,in = 854.5\ in^3$$

3. Subtract the volumes.

$$3418.1\,in^3 - 854.5\ in^3 \approx 2563.6\,in^3$$

The volume of the solid portion of pipe is about $2563.6\,in^3$.