Objective: Write a polynomial function of least degree given the complex roots.

Concept

Complex Conjugate Root Theorem

If P is a polynomial equation in one variable with real coefficients, and $a + bi$ is a root of P, with a and b real numbers, then its complex conjugate $a - bi$ is also a root of P. Also, if $a + \sqrt{b}$ is a root, so is its conjugate $a - \sqrt{b}$.

For example:

If $3 + \sqrt{2}$ is a root, then $3 - \sqrt{2}$ is a root.

If $-1 - 3i$ is a root, then $-1 + 3i$ is a root.

If $-\sqrt{5}$ is a root, then $\sqrt{5}$ is a root.

If $2i$ is a root, then $-2i$ is a root.

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Concept

Steps to write a polynomial function using the given roots.

1. Determine any missing irrational or imaginary roots using the Complex Conjugate Root Theorem.
zeros

2. Find the factors related to each root.

3. Multiply the factors together to write the function $p(x)$ in standard form.

Objective: Write a polynomial function of least degree given the complex roots.

Ex) Write a polynomial function, $p(x)$, of least degree in standard form using the given roots.

$$roots: \sqrt{2}, -4i$$

irrational imaginary

① find missing roots/zeros

- have $^*\sqrt{2}$ → also have $^*-\sqrt{2}$
- have $^*-4i$ → also have $^*4i$

} 4 roots/zeros

② find the factors

$x = \sqrt{2}$, $-\sqrt{2}$ $-\sqrt{2}$, $x - \sqrt{2} = 0$ factor

or $x = -\sqrt{2}$, $+\sqrt{2}$ $+\sqrt{2}$, $x + \sqrt{2} = 0$ factor

or $x = -4i$, $+4i$ $+4i$, $x + 4i = 0$ factor

or $x = 4i$, $-4i$ $-4i$, $x - 4i = 0$ factor

③ factored form

$$p(x) = (x - \sqrt{2})(x + \sqrt{2})(x + 4i)(x - 4i)$$

④ multiply the factors

$x^2 + \cancel{x\sqrt{2}} \overset{0}{\cancel{- x\sqrt{2}}} - \sqrt{4}$

$(x^2 - 2)$

$x^2 \cancel{- 4xi} \overset{0}{\cancel{+ 4xi}} - 16i^2$

$-16(-1)$

$(x^2 + 16)$

$$p(x) = (x^2 - 2)(x^2 + 16)$$

$$x^4 + 16x^2 - 2x^2 - 32$$

⑤ standard form

$$\boxed{p(x) = x^4 + 14x^2 - 32}$$

<u>Objective</u>: Write a polynomial function of least degree given the complex roots.

Ex) Write a polynomial function, $p(x)$, of least degree in standard form using the given roots.

$$roots: -6, 5, and\ \frac{3}{5}$$

* all rational roots

① find missing roots/zeros

* no missing roots

② find the factors

$x = -6$, $+6$ $+6$: $x + 6 = 0$ factor

or $x = 5$, -5 -5: $x - 5 = 0$ factor

or $5 \cdot x = \frac{3}{\cancel{5}} \cdot \cancel{5}^1$

$5x = 3$, -3 -3: $5x - 3 = 0$ factor

③ factored form

$p(x) = (x+6)(x-5)(5x-3)$

④ multiply the factors

$x^2 - 5x + 6x - 30$

$(x^2 + x - 30)(5x - 3)$

$(5x - 3)(x^2 + x - 30)$

$5x^3 + 5x^2 - 150x$

$\quad - 3x^2 - 3x + 90$

⑤ standard form

$$\boxed{p(x) = 5x^3 + 2x^2 - 153x + 90}$$

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Closure

If one root of a fourth degree polynomial function is rational, what must be true about the other three roots?

One of the other three roots must also be rational. The other two roots could be rational or either irrational conjugates or imaginary conjugates.