Objective: Write a quadratic function in vertex form by completing the square.

Concept

A perfect square trinomial is a trinomial that can be factored into a binomial squared.

Examples	Non-Examples
$x^2 - 4x + 4 = (x - 2)^2$	$x^2 - 4x + 3 = (x - 3)(x - 1)$
$x^2 + 10x + 25 = (x + 5)^2$	$x^2 + 10x + 16 = (x + 2)(x + 8)$

Vertex Form of a Quadratic Function contains a perfect square trinomial in factored form.

Vertex Form: $f(x) = a(x - h)^2 + k$

$f(x) = -2(x - 2)^2 + 5$ $\qquad$ $f(x) = 3(x + 1)^2 - 6$

In order to rewrite standard form, $f(x) = ax^2 + bx + c$ in vertex form, $f(x) = a(x - h)^2 + k$, we must learn how to create a perfect square trinomial.

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Consider the two perfect square trinomials below. What is a mathematical relationship between the values of b and c in both trinomials? In other words, how could you use the value of b to get the value of c?

$x^2 + 10x + 25$ $\qquad$ $x^2 - 4x + 4$

I could take the value of b, divide it by 2, and then square the number to find c.

$\left(\frac{10}{2}\right)^2 = 25$ $\qquad$ $\left(\frac{-4}{2}\right)^2 = 4$

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Rewrite the trinomial $x^2 + 6x + 10$ in the form $(x + 3)^2 + \underline{1}$.

① $x^2 + 6x + \underline{9} + 10 - \underline{9}$

$\left(\frac{6}{2}\right)^2 = 3^2$

$(x+3)(x+3) + 1$

$(x+3)^2$

Rewrite the trinomial $x^2 - 8x - 3$ in the form $(x + -4)^2 + \underline{-19}$.

① $x^2 - 8x + \underline{16} - 3 - \underline{16}$

$\left(\frac{-8}{2}\right)^2 = (-4)^2$

$-3 + -16$

$(x-4)(x-4)$

$(x-4)^2 - 19$

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Steps to Create Vertex Form, $f(x) = a(x-h)^2 + k$, from Standard Form, $f(x) = ax^2 + bx + c$, by Completing the Square

1. **Move the constant term to the function side:**
$$f(x) = ax^2 + bx + c \rightarrow f(x) - c = ax^2 + bx$$
2. **Factor out the value of a:** $f(x) - c = ax^2 + bx \rightarrow f(x) - c = a(x^2 + dx)$
3. **Create a perfect square trinomial. Make sure you balance the equation by adding to both sides.**
$$f(x) - c = a(x^2 + dx) \rightarrow f(x) - c + a\left(\frac{d}{2}\right)^2 = a(x^2 + dx + \left(\frac{d}{2}\right)^2)$$
4. **Simplify the left side and write the perfect square trinomial as a binomial squared:** $f(x) - k = a(x-h)^2$
5. **Write the function in vertex form:** $f(x) = a(x-h)^2 + k$

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Ex) Write each quadratic function in vertex form. Then identify the vertex and if it is a maximum or minimum.

$$f(x) = -x^2 + 6x - 5$$

① $+5 \qquad +5$

$f(x) + 5 = -1x^2 + 6x$

② factor out a

$f(x) + 5 = -1(x^2 - 6x)$

③ $f(x) + 5 + \frac{-9}{9 \cdot -1} = -1(x^2 - 6x + \frac{9}{(\frac{-6}{2})^2})$

$(-3)^2$

④ $f(x) - 4 = -1(x - 3)^2$

⑤ $+4 \qquad +4$

$f(x) = -(x-3)^2 + 4$

vertex form

Vertex $(3, 4)$; maximum

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Ex) Write each quadratic function in vertex form. Then identify the vertex and if it is a maximum or minimum.

$$f(x) = 3x^2 + 24x - 3$$

① $+3 \qquad +3$

$f(x) + 3 = 3x^2 + 24x$

② factor out a 3: $f(x) + 3 = 3(x^2 + 8x)$

③ $f(x) + 3 + 48 = 3(x^2 + 8x + 16)$

$16 \cdot 3$ $\qquad \left(\frac{8}{2}\right)^2 = (4)^2$

④ $f(x) + 51 = 3(x + 4)^2$

$-51 \qquad -51$

⑤ $f(x) = 3(x+4)^2 - 51$

vertex form

vertex $(-4, -51)$; minimum

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Ex) Write each quadratic function in vertex form. Then identify the vertex and if it is a maximum or minimum.

$$f(x) = x^2 - 10x - 8$$

① $+8 \qquad +8$

$f(x) + 8 = 1x^2 - 10x$

② factor out a

$f(x) + 8 = 1(x^2 - 10x)$

③ $f(x) + 8 + \frac{25}{25 \cdot 1} = 1(x^2 - 10x + \frac{25}{(\frac{-10}{2})^2})$

$(x-5)(x-5)$ $\qquad (-5)^2$

④ $f(x) + 33 = 1(x-5)^2$

$-33 \qquad -33$

⑤ $f(x) = (x-5)^2 - 33$ vertex form

Vertex $(5, -33)$; minimum

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Closure

Michelle rewrote the standard form of a quadratic function in vertex form. Her work is shown. What error did Michelle make? If you fix her mistake, what is the correct vertex form?

standard form: $f(x) = 2x^2 - 16x - 8$

step 1: $f(x) + 8 = 2x^2 - 16x$

step 2: $f(x) + 8 = 2\left(x^2 - 16x\right)$

step 3: $f(x) + 8 + 128 = 2\left(x^2 - 16x + 64\right)$

step 4: $f(x) + 136 = 2(x - 8)^2$

vertex form: $f(x) = 2(x - 8)^2 - 136$

Michelle's error is that she forgot to factor 2 out of 16, making the new number an 8. If I fix her mistake, the correct vertex form is $f(x) = 2(x - 4)^2 - 40$.