

Objective: Create rational functions using operations.

Concept

One way to create functions is to combine two or more functions using a basic operation.

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$



Objective: Create rational functions using operations.

Ex) Given the functions  $f(x) = \frac{x+2}{x}$  and  $g(x) = \frac{4x-1}{x-1}$ , create the following function.

$$\begin{aligned}
 (f - g)(x) &= f(x) - g(x) \\
 &= \frac{x+2}{x} - \frac{4x-1}{x-1} \quad \text{LCD} = x(x-1) \\
 &= \frac{(x+2) \cdot \frac{(x-1)}{(x-1)}}{x \cdot \frac{(x-1)}{(x-1)}} - \frac{(4x-1) \cdot \frac{x}{x}}{(x-1) \cdot \frac{x}{x}} \\
 &= \frac{x^2+x-2}{x(x-1)} - \frac{4x^2-x}{x(x-1)} \\
 &= \frac{x^2+x-2-4x^2+x}{x(x-1)}
 \end{aligned}$$

$$(f - g)(x) = \frac{-3x^2 + 2x - 2}{x^2 - x}$$



Objective: Create rational functions using operations.

Ex) Given the functions  $f(x) = \frac{x+4}{x^2-4}$  and  $g(x) = \frac{x-2}{x+2}$ , create the following function.

$$(f+g)(x) = f(x) + g(x)$$

$$= \frac{x+4}{x^2-4} + \frac{x-2}{(x+2)}$$

$LCD = (x+2)(x-2)$

$$= \frac{(x+4)}{(x+2)(x-2)} \cdot \frac{1}{1} + \frac{(x-2)}{(x+2)} \cdot \frac{(x-2)}{(x-2)}$$

$$= \frac{x+4}{(x+2)(x-2)} + \frac{x^2 - 4x + 4}{(x+2)(x-2)}$$

$$= \frac{x+4 + x^2 - 4x + 4}{(x+2)(x-2)}$$

$$(f+g)(x) = \frac{x^2 - 3x + 8}{x^2 - 4}$$

Objective: Create rational functions using operations.

Ex) Given the functions  $f(x) = \frac{x^2-4x-12}{x^2-9}$  and  $g(x) = \frac{x^2+3x}{2x+4}$ , create the following function.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= \frac{x^2-4x-12}{x^2-9} \cdot \frac{x^2+3x}{2x+4}$$

$$= \frac{(x-6)\cancel{(x+2)}}{\cancel{(x+3)}(x-3)} \cdot \frac{x\cancel{(x+3)}}{2\cancel{(x+2)}}$$

$$= \frac{x(x-6)}{2(x-3)}$$

$$(f \cdot g)(x) = \frac{x^2-6x}{2x-6}$$

Objective: Create rational functions using operations.

Ex) Given the functions  $f(x) = \frac{x^2+7x+6}{x^2+x-6}$  and  $g(x) = \frac{x^2+3x+2}{x^2-4}$ , create the following function.

$$\begin{aligned}
 \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} = f(x) \div g(x) \\
 &= \frac{x^2+7x+6}{x^2+x-6} \div \frac{x^2+3x+2}{x^2-4} \\
 &= \frac{x^2+7x+6}{x^2+x-6} \cdot \frac{x^2-4}{x^2+3x+2} \\
 &= \frac{(x+6)\cancel{(x+1)}}{(x+3)\cancel{(x-2)}} \cdot \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}\cancel{(x+1)}} \\
 \left(\frac{f}{g}\right)(x) &= \frac{x+6}{x+3}
 \end{aligned}$$