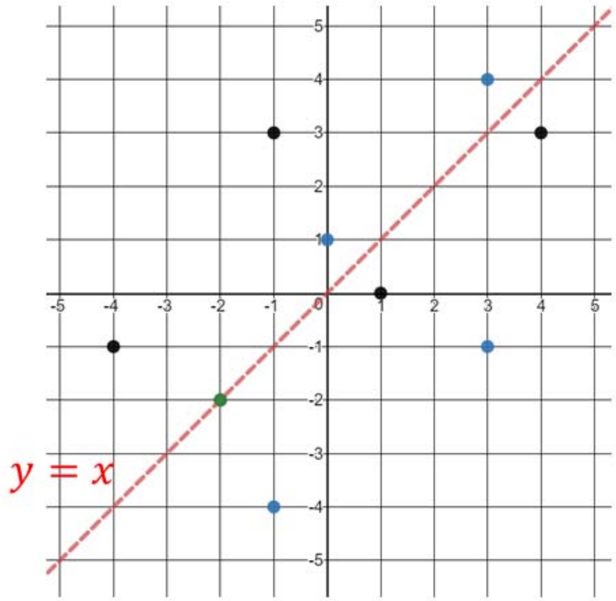
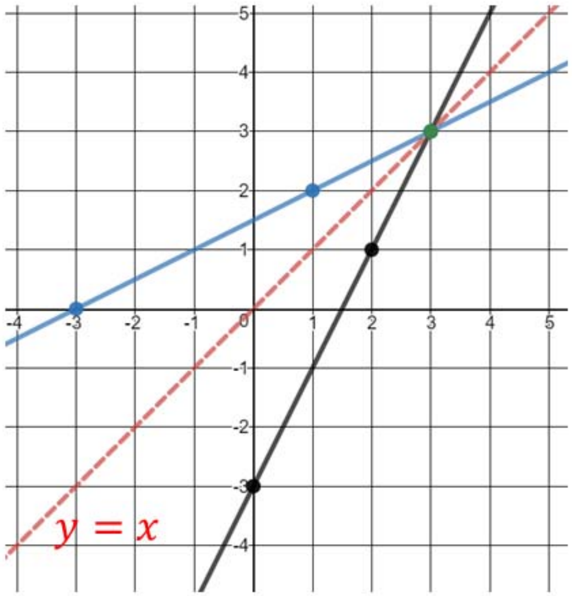
Objective: Graph a function and its inverse

Concept

The graph of a function or relation and its inverse will always be reflections of each other over the line $y = x$. All points of intersection between a function or relation and its inverse will be on the line $y = x$.





Objective: Graph a function and its inverse

Steps to Graph the Inverse from the Graph of a Function

1. Graph the line of reflection, $y = x$, as a dashed line.
2. Find the coordinates of at least three points on the graph of the function.
3. Interchange (switch) the coordinates of these points to find points on the inverse.
4. Graph these points and draw a line or curve through them. Make sure that any points of intersection with the function are on the line $y = x$.
5. Label the graph of the inverse using inverse notation.

Objective: Graph a function and its inverse

Ex) Graph the function $f(x) = \frac{1}{2}x + 3$ and $f^{-1}(x)$.

① Graph $f(x)$

$f(x) = \frac{1}{2}x + 3$

$mx + b$

☆ linear

y-int (0, 3)

slope = $\frac{1}{2}$

② graph $y = x$ $mx+b$

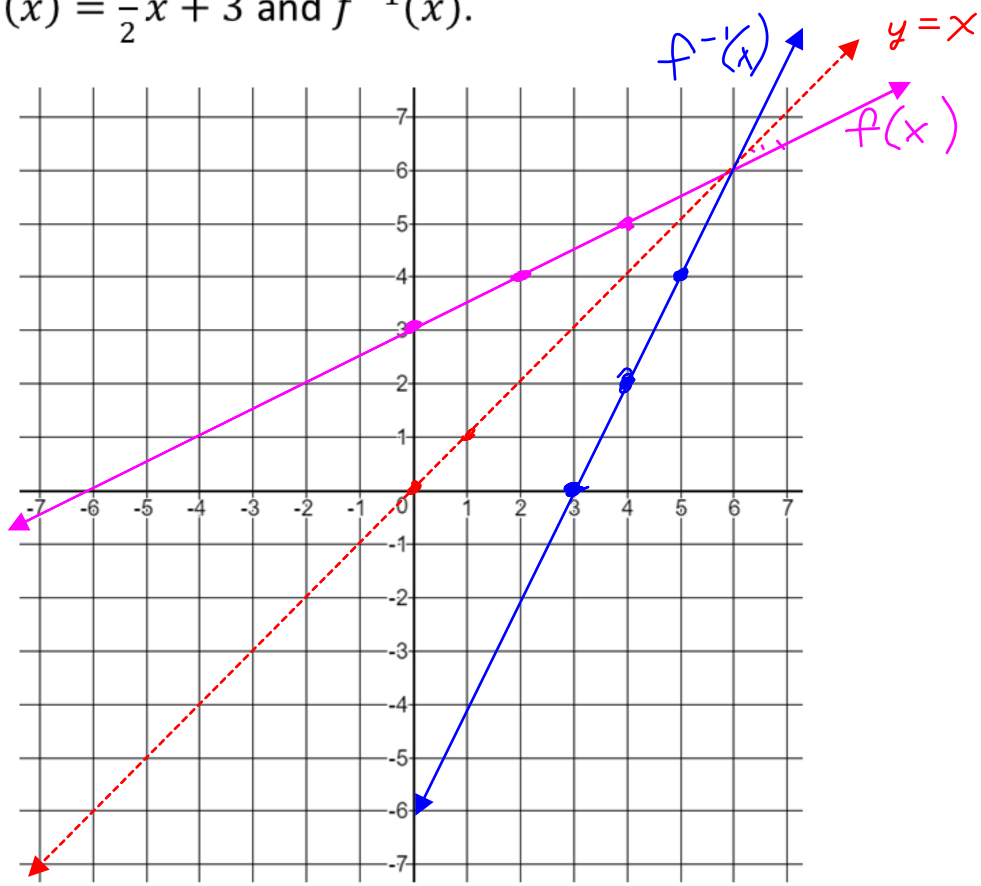
☆ dashed line

y-in (0,0)

slope = 1 = $\frac{1}{1}$

③ graph $f^{-1}(x)$

$f(x)$	$f^{-1}(x)$
(0, 3) →	(3, 0)
(2, 4) →	(4, 2)
(4, 5) →	(5, 4)



Objective: Graph a function and its inverse

Ex) Graph the function $h(x) = -3x - 1$ and $h^{-1}(x)$.

① graph $h(x)$

$h(x) = -3x - 1$

$mx + b$

☆ linear

y-int $(0,-1)$

slope $= -3 = \frac{-3 \downarrow}{1 \rightarrow}$

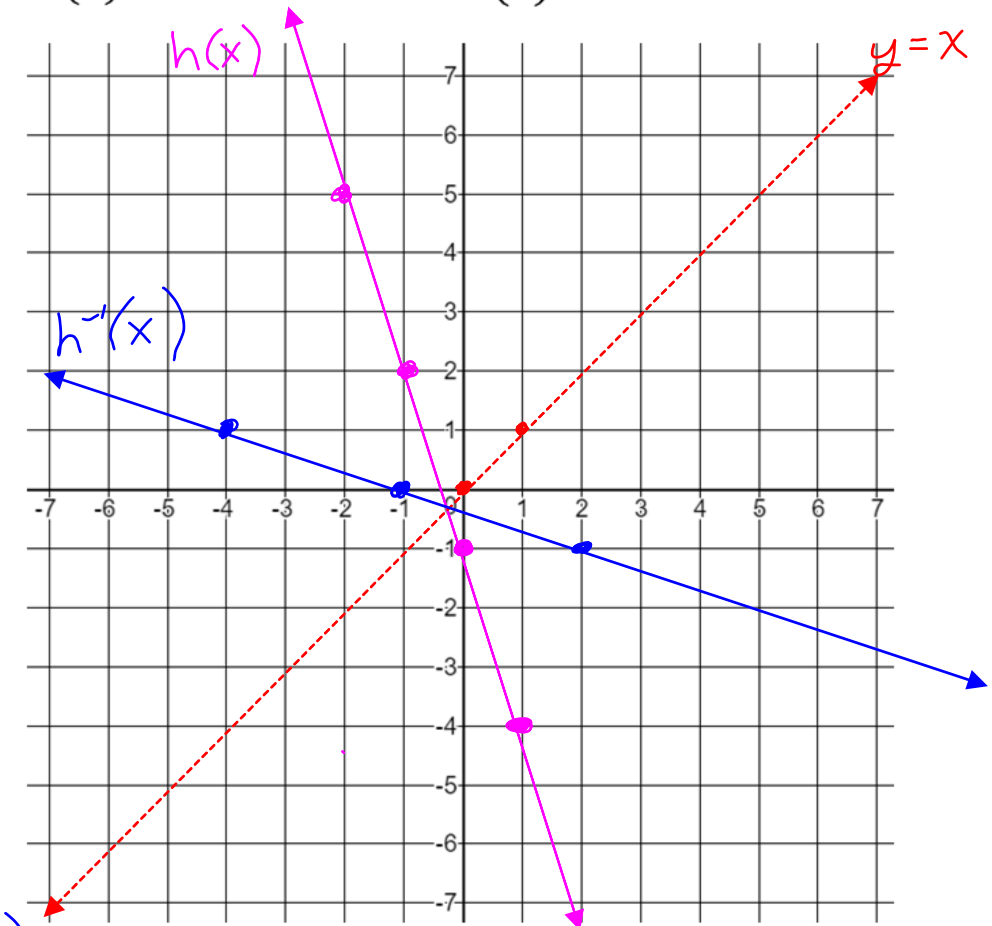
$= \frac{3 \uparrow}{-1 \leftarrow}$

② graph $y = x$

☆ dashed line

③ graph $h^{-1}(x)$

$h(x)$		$h^{-1}(x)$
$(0,-1)$	$\rightarrow$	$(-1,0)$
$(1,-4)$	$\rightarrow$	$(-4,1)$
$(-1,2)$	$\rightarrow$	$(2,-1)$



Objective: Graph a function and its inverse

Concept

The domain of the quadratic function $f(x) = x^2$ is restricted in order to create an inverse that is a function.
If the domain is restricted to $x \geq 0$ then the inverse function is $f^{-1}(x) = \sqrt{x}$.
If the domain is restricted to $x \leq 0$ then the inverse function is $f^{-1}(x) = -\sqrt{x}$.

Objective: Graph a function and its inverse

Ex) Graph the function $f(x) = x^2$ for $x \geq 0$ and $f^{-1}(x)$.

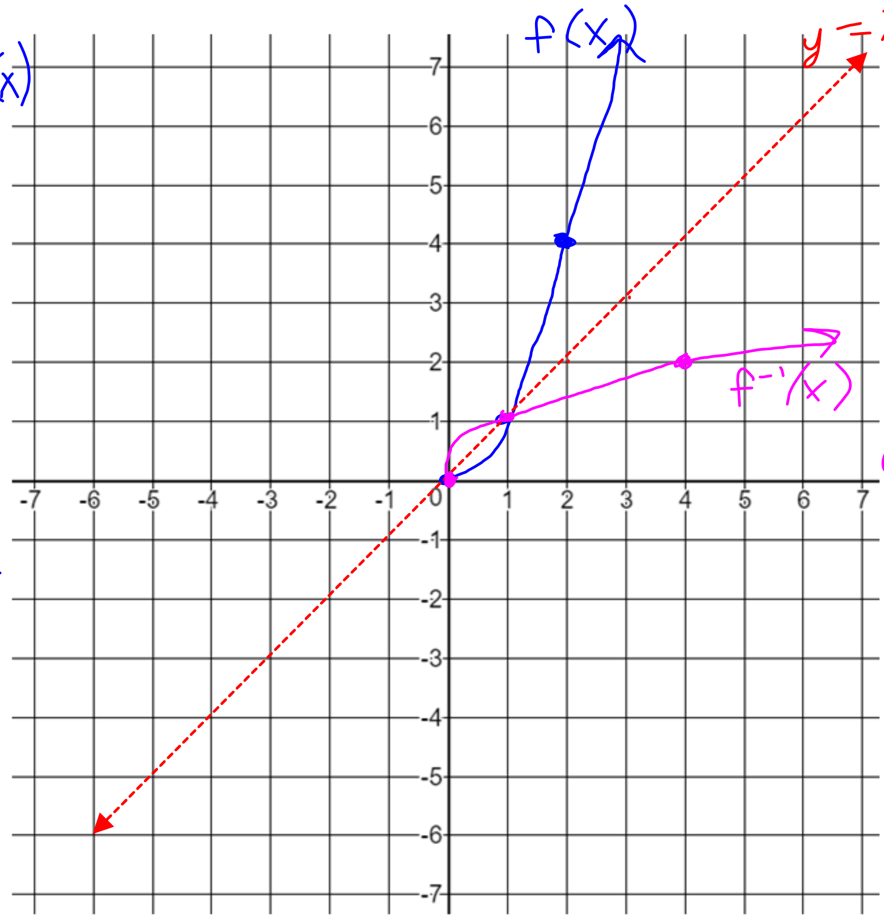
① graph f(x)

$f(x) = x^2$

$x \geq 0$

☆ quadratic
part of a
parabola

x	$f(x)=x^2$
0	0
1	1
2	4



② graph
y = x
☆ dashed

③ graph
$f^{-1}(x)$

$f(x)$		$f^{-1}(x)$
(0,0)	$\rightarrow$	(0,0)
(1,1)	$\rightarrow$	(1,1)
(2,4)	$\rightarrow$	(4,2)

Objective: Graph a function and its inverse

Ex) Graph the function $g(x) = 2x^2 - 2$ for $x \geq 0$ and $g^{-1}(x)$.

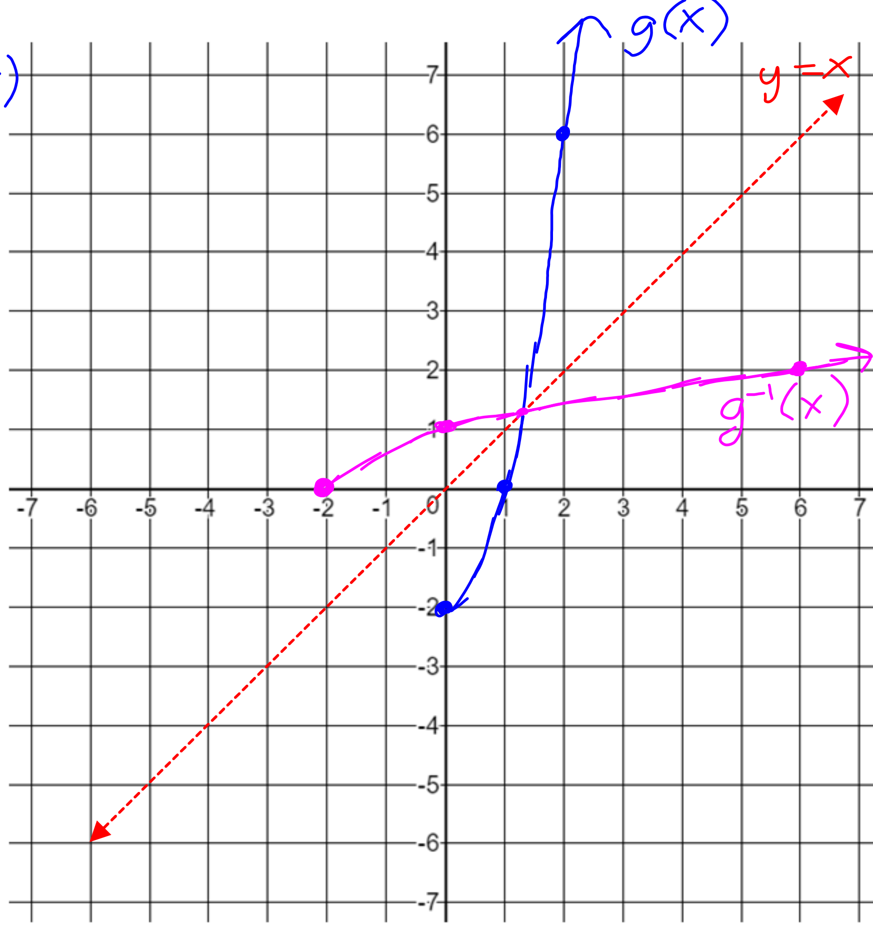
① graph $g(x)$

$g(x) = 2x^2 - 2$

$x \geq 0$

* part of a parabola

x	$g(x)$
0	−2
1	0
2	6



② graph $y=x$

* dashed line

③ graph $g^{-1}(x)$

$g(x)$		$g^{-1}(x)$
(0,−2)	→	(−2,0)
(1,0)	→	(0,1)
(2,6)	→	(6,2)

Objective: Graph a function and its inverse

Ex) Graph the function $d(x) = (x+2)^2$ for $x \geq -2$ and $d^{-1}(x)$.

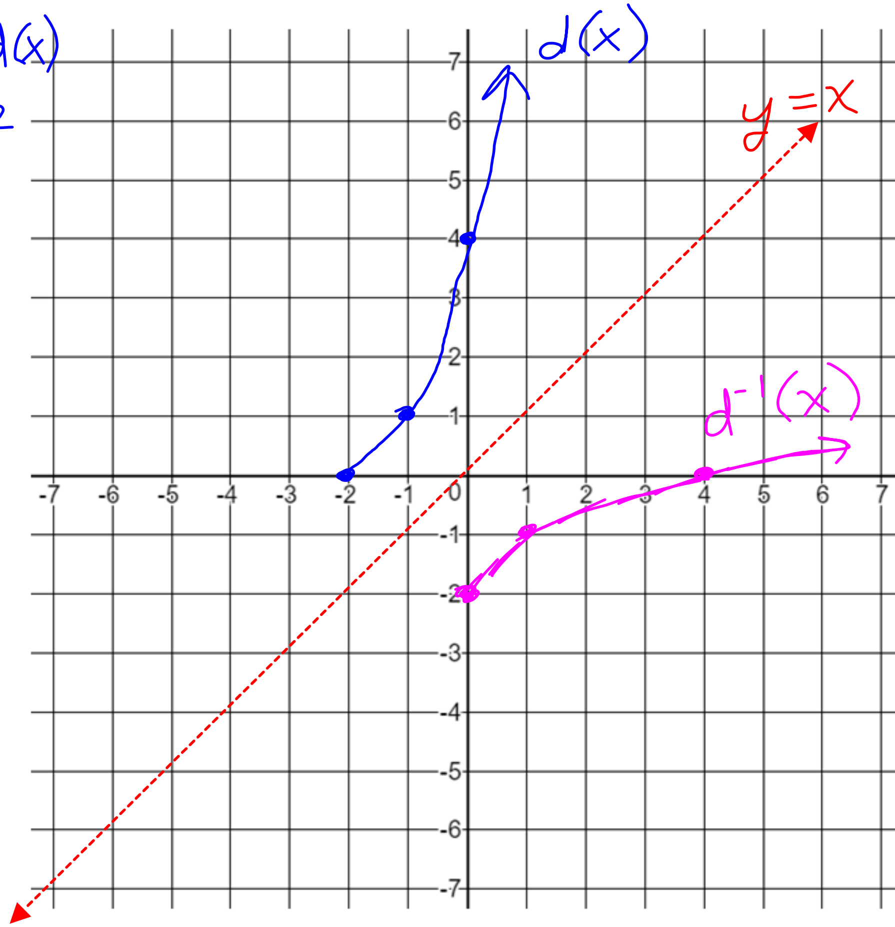
① graph d(x)

$d(x) = (x+2)^2$

$x \geq -2$

* part of a parabola

x	$d(x)$
-2	0
-1	1
0	4



② graph $y = x$

* dashed line

③ graph $d^{-1}(x)$

$d(x)$		$d^{-1}(x)$
$(-2, 0)$	$\rightarrow$	$(0, -2)$
$(-1, 1)$	$\rightarrow$	$(1, -1)$
$(0, 4)$	$\rightarrow$	$(4, 0)$

Objective: Graph a function and its inverse

Concept

The inverse of the cubic function $f(x) = x^3$ is the cube root function $f^{-1}(x) = \sqrt[3]{x}$. The domain of the cubic function $f(x) = x^3$ does not need to be restricted in order to create an inverse that is a function because

- the cube root of a nonnegative real number is always a unique nonnegative real number
- and the cube root of a negative real number is always a unique negative real number.
- Therefore, for each value of the domain there will be exactly one range value.

Objective: Graph a function and its inverse

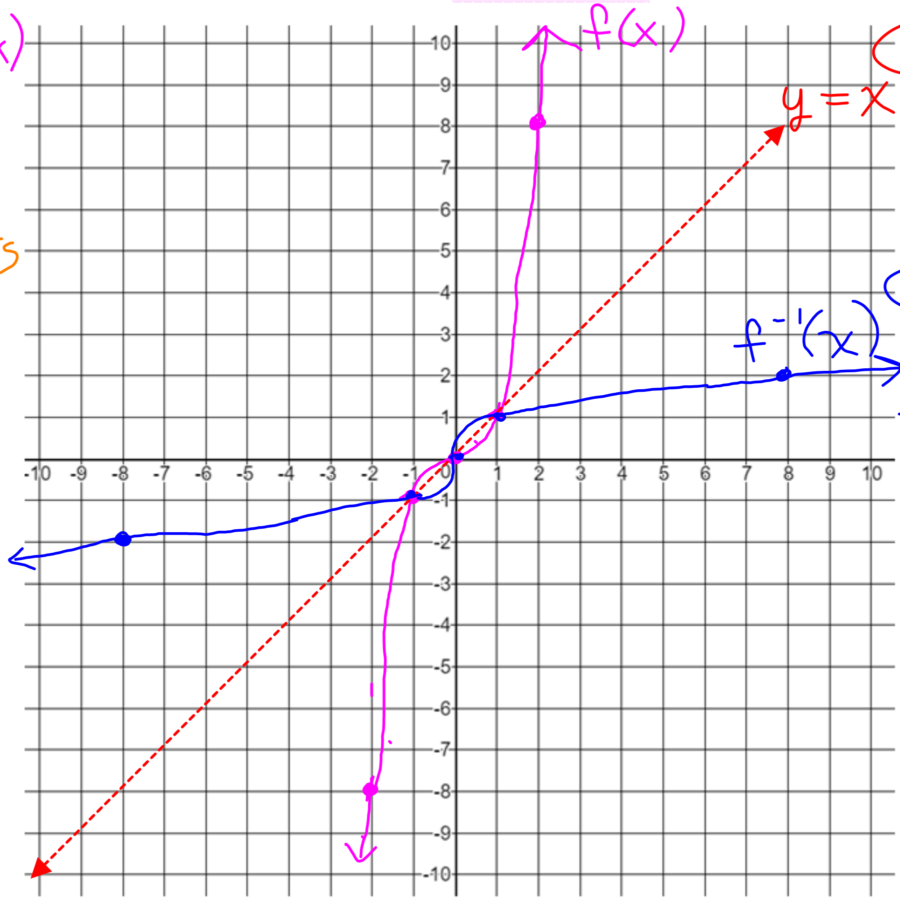
Ex) Graph the function $f(x) = x^3$ and $f^{-1}(x)$.

① graph f(x)

$f(x) = x^3$

* need 5 points

x	$f(x) = x^3$
2	8
1	1
0	0
−1	−1
−2	−8



② graph y = x

* dashed line

③ graph $f^{-1}(x)$

$f(x)$		$f^{-1}(x)$
(2, 8)	→	(8, 2)
(1, 1)	→	(1, 1)
(0, 0)	→	(0, 0)
(−1, −1)	→	(−1, −1)
(−2, −8)	→	(−8, −2)

Objective: Graph a function and its inverse

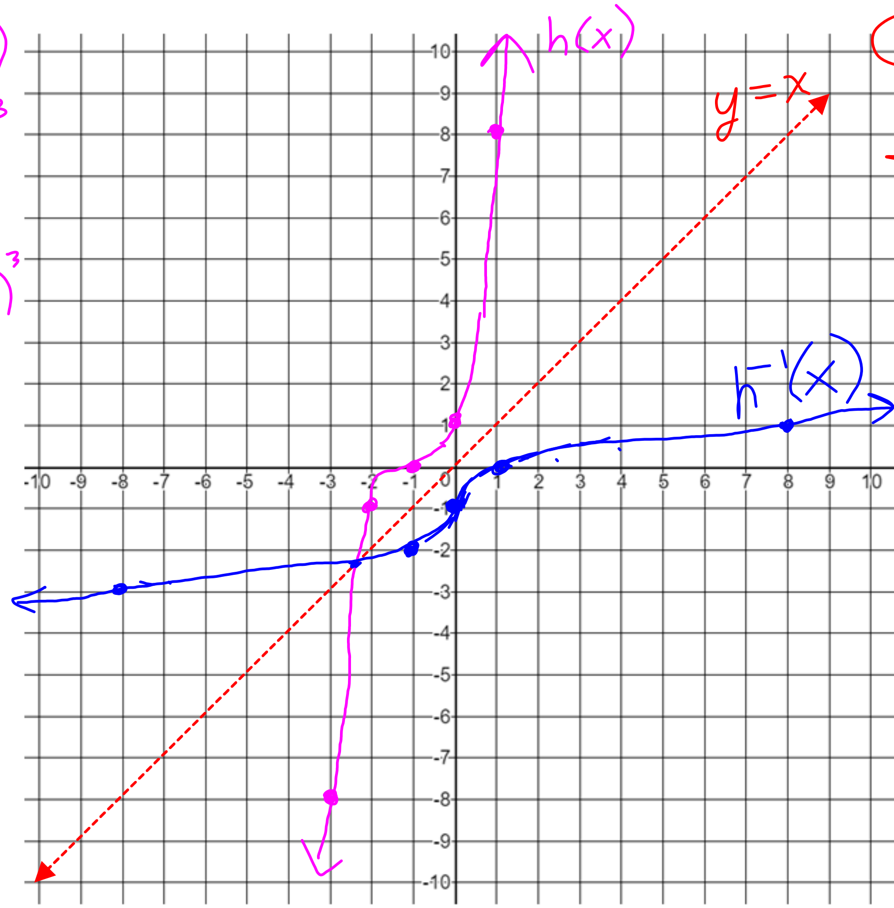
Ex) Graph the function $h(x) = (x+1)^3$ and $h^{-1}(x)$.

① graph $h(x)$

$h(x) = (x+1)^3$

* $h = -1$

x	$h(x) = (x+1)^3$
1	8
0	1
−1	0
−2	−1
−3	−8



② graph $y = x$

* dashed line

③ graph $h^{-1}(x)$

$h(x)$		$h^{-1}(x)$
$(1, 8)$	$\rightarrow$	$(8, 1)$
$(0, 1)$	$\rightarrow$	$(1, 0)$
$(-1, 0)$	$\rightarrow$	$(0, -1)$
$(-2, -1)$	$\rightarrow$	$(-1, -2)$
$(-3, -8)$	$\rightarrow$	$(-8, -3)$

Objective: Graph a function and its inverse

Closure

True or False: The domain of a function is always restricted in order to create an inverse that is a function. Explain.

False, only the domain of some functions, like the quadratic function, must be restricted for the inverse to be a function.