Objective: Graph cube root functions using transformations.

Concept

For cube root functions $f(x) = a\sqrt[3]{x-h}+k$ and $g(x) = \sqrt[3]{\frac{1}{b}(x-h)}+k$

the transformations are:

translation h units left or right

translation k units up or down

reflection across the x-axis when $a < 0$

vertical stretch when $|a| > 1$

vertical compression when $|a| < 1$

reflection across the y-axis when $b < 0$

horizontal stretch when $|b| > 1$ (i.e. if $\frac{1}{\mathrm{b}} = \frac{1}{2} \rightarrow b = 2$)

horizontal compression when $|b| < 1$ (i.e. if $\frac{1}{\mathrm{b}} = 2 \rightarrow b = \frac{1}{2}$)

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Ex) For each function, describe the effect of each parameter on the parent function $f(x) = \sqrt[3]{x}$.

$$g(x) = -\frac{3}{5}\sqrt[3]{x-4}+5$$

$a = \frac{-3}{5}$ x-axis refl.

$|a| = \left|-\frac{3}{5}\right| = \frac{3}{5} < 1$ vert. comp

$h = 4$ right 4

$k = 5$ up 5

The effects of the parameters compared to $f(x) = \sqrt[3]{x}$ are an x-axis reflection, a vertical compression by a factor of $\frac{3}{5}$ and a translation right 4 units and up 5 units.

$$g(x) = \sqrt[3]{\left(-\frac{1}{5}\right)(x+7)} - 2$$

$1/b$

$\frac{1}{b} = -\frac{1}{5} \rightarrow b = -5$ y-axis refl.

$|b| = |-5| = 5 > 1$ horiz. stretch

$h = -7$ left 7

$k = -2$ down 2

The effects of the parameters compared to $f(x) = \sqrt[3]{x}$ are a y-axis reflection, a horizontal stretch by a factor of 5, and a translation left 7 units and down 2 units.

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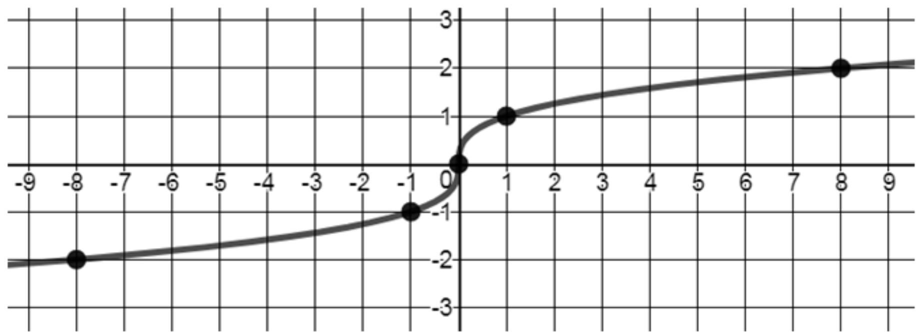
The cube root parent function is $f(x) = \sqrt[3]{x}$. Since the cube root of a negative real number is a negative real number, there is no need to restrict the domain of this function as there is with the square root function.

x	$f(x) = \sqrt[3]{x}$
-8	-2
-1	-1
0	0
1	1
8	2



Domain: $\{x | -\infty < x < +\infty\}$; $(-\infty, +\infty)$

Range: $\{y | -\infty < y < +\infty\}$; $(-\infty, +\infty)$

End Behavior: $as\ x \to -\infty, f(x) \to -\infty$
$as\ x \to +\infty, f(x) \to +\infty$

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Concept

One Procedure for Graphing a Cubic Function Using Transformations

1. Determine the translations and graph the translation of $(0,0)$. This point will not be affected by any reflection/stretch/compression.
2. For a parameter of a, draw a dashed horizontal line through the transformed point from step 1. For a parameter of $\frac{1}{b}$, draw a vertical line through the transformed point from step 1.
3. Perform any reflection, stretch, and/or compression on the other key points in the parent function using the line in step 2 as the reference line.
4. Draw in a smooth curve. Erase the dashed line from step 2.

Objective: Graph cube root functions using transformations.

Ex) A) Graph the function using transformations. B) State the domain and range using set and interval notation. C) State the end behavior.

$$g(x) = -2\sqrt[3]{x-3} + 5$$

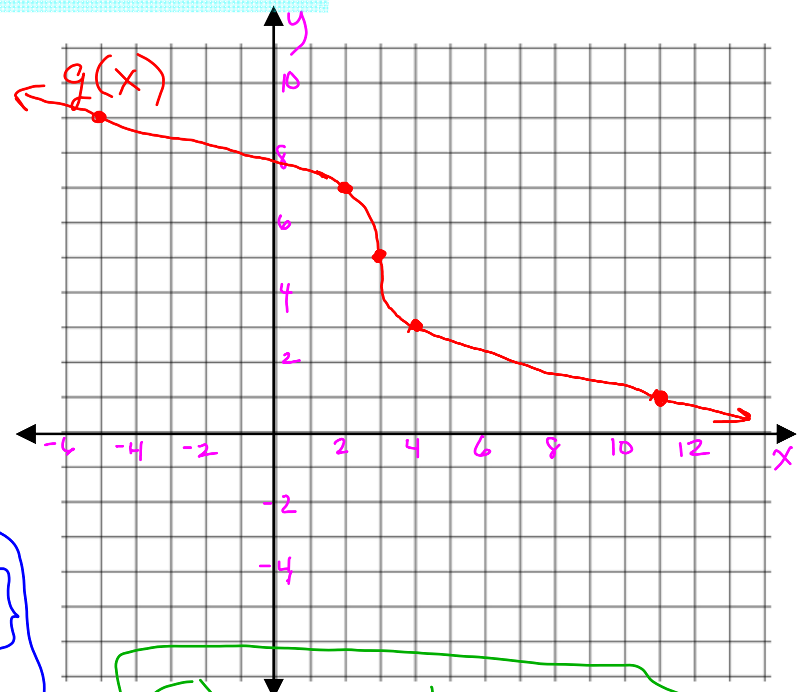
a

$a = -2$ x-axis refl.

$|a| = |-2| = 2 > 1$ vert. stretch

$h = 3$ right 3

$k = 5$ up 5



(B) Domain: $\{x \mid -\infty < x < \infty\}$ set

interval $(-\infty, \infty)$

Range: $\{y \mid -\infty < y < \infty\}$ set

$(-\infty, \infty)$ interval

(C) end behavior

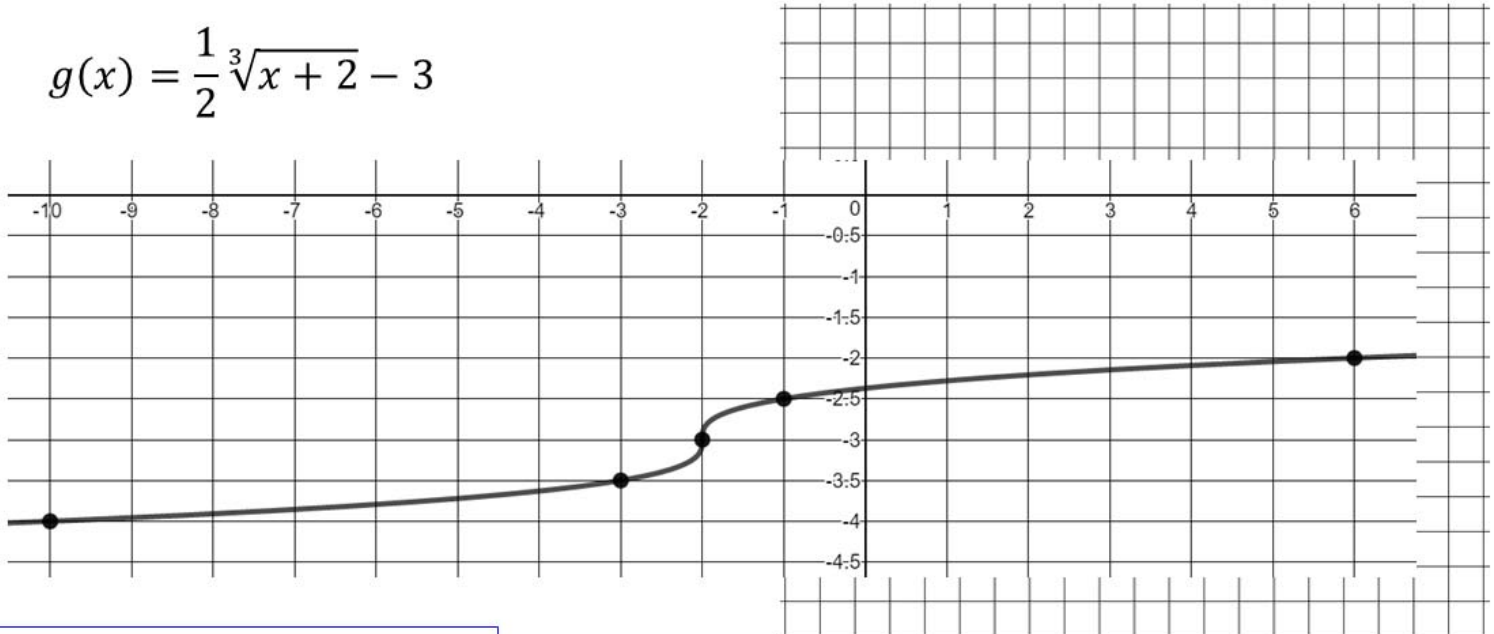
as $x \to -\infty$, $g(x) \to +\infty$

as $x \to +\infty$, $g(x) \to -\infty$

Objective: Graph cube root functions using transformations.

Practice) A) Graph the function using transformations. B) State the domain and range using set and interval notation. C) State the end behavior.

$$g(x) = \frac{1}{2}\sqrt[3]{x+2} - 3$$



Domain: $\{x | -\infty < x < \infty\}$
$(-\infty, +\infty)$

Range: $\{y | -\infty < y < \infty\}$
$(-\infty, +\infty)$

$as\ x \to -\infty, g(x) \to -\infty$
$as\ x \to +\infty, g(x) \to +\infty$

Objective: Graph cube root functions using transformations.

Ex) A) Graph the function using transformations. B) State the domain and range using set and interval notation. C) State the end behavior.

$$g(x) = \sqrt[3]{-2(x-1)} + 3$$

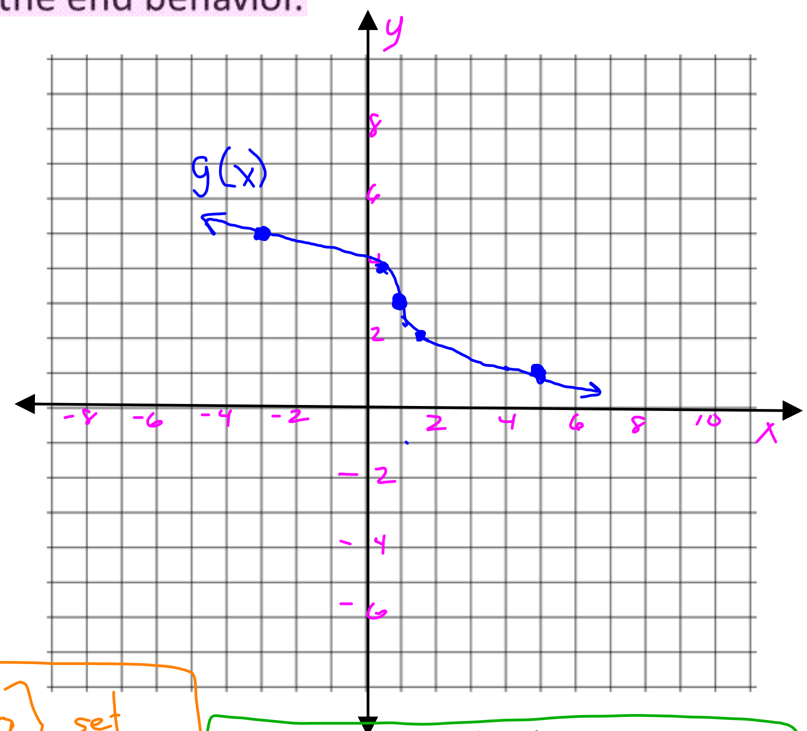
$-\frac{1}{b}$

$\frac{1}{b} = -2 \rightarrow b = -\frac{1}{2}$ y-axis refl.

$|b| = \left|\frac{-1}{2}\right| = \frac{1}{2} < 1$ horiz. comp.

$h = 1$ right 1

$k = 3$ up 3



(B) Domain: $\{x \mid -\infty < x < \infty\}$ set

$(-\infty, \infty)$ interval

Range: $\{y \mid -\infty < y < \infty\}$ set

$(-\infty, \infty)$ interval

(C) end behavior

as $x \to -\infty$, $g(x) \to +\infty$

as $x \to +\infty$, $g(x) \to -\infty$

Objective: Graph cube root functions using transformations.

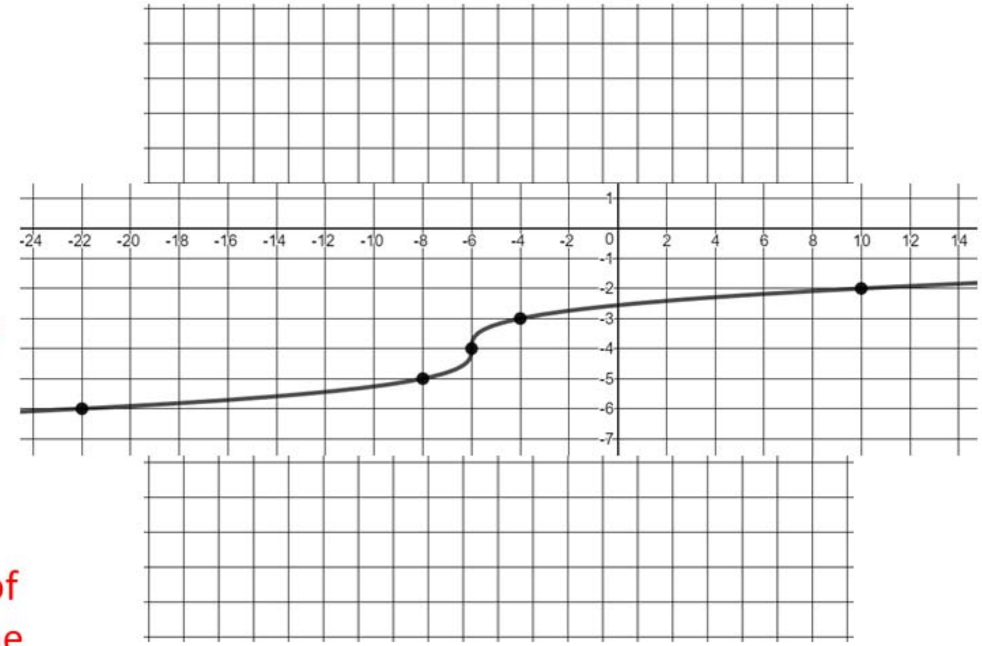
Practice) A) Graph the function using transformations. B) State the domain and range using set and interval notation. C) State the end behavior.

$$g(x) = \sqrt[3]{\frac{1}{2}(x+6)} - 4$$

$b = 2, \qquad h = -6, \qquad k = -4$

Translating the point $(0,0)$ creates the point $(-6,-4)$. This point will not be affected by any stretch/compression/reflection.

Now use the line $x = -6$ as the line of reflection and the reference line for the horizontal stretch by a factor of 3.



Domain: $\{x \mid -\infty < x < \infty\}$
$(-\infty, +\infty)$

Range: $\{y \mid -\infty < y < \infty\}$
$(-\infty, +\infty)$

$as\ x \to -\infty, g(x) \to -\infty$
$as\ x \to +\infty, g(x) \to +\infty$

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Closure

Patricia found the domain of $f(x) = \sqrt[3]{x+7} - 3$. Her work is shown. Do you agree or disagree with her answer? Explain your reasoning.

$$x + 7 \geq 0$$
$$\underline{\;-7 \quad -7\;}$$
$$x \geq -7$$
$$\boxed{Domain: \{x \mid x \geq -7\}; [-7, \infty)}$$

I disagree with Patricia's answer. The function $f(x) = \sqrt[3]{x+7} - 3$ is a cube root function, and the domain does not need to be restricted to non-negative values. The domain of all cube root functions is the set of all real numbers.