Objective: Graph Logarithmic Functions and State Key Features

## Concept

## One Way to Graph Logarithmic Functions

1. Write the powers of the base of the logarithm.
2. The exponents (logarithms) will be the $y$-values of the points of the logarithmic function. The values of the powers will be the $x$-values of the points of the logarithmic function. (power, exponent)
3. Graph these points, applying any transformations for the function.

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## Concept

The Base 2 Logarithm Function

$$
f(x)=\log _{2} x
$$



$$
y=\log _{2} x \rightarrow 2^{y}=x
$$

| Powers of 2 | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: | :---: |
| $2^{-1}=\frac{1}{2}$ | $\frac{1}{2}$ | -1 |
| $2^{0}=1$ | 1 | 0 |
| $2^{1}=2$ | 2 | 1 |
| $2^{2}=4$ | 4 | 2 |
| $2^{3}=8$ | 8 | 3 |

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## Concept

The Base 10 Logarithm Function

$y=\log x \rightarrow \mathbf{1 0}^{\boldsymbol{y}}=\boldsymbol{x}$

| Powers of $\mathbf{1 0}$ | $x$ | $y$ |
| :---: | :---: | :---: |
| $10^{-1}=\frac{1}{10}$ | $\frac{1}{10}$ | -1 |
| $10^{0}=1$ | 1 | 0 |
| $10^{1}=10$ | 10 | 1 |
| $10^{2}=100$ | 100 | 2 |

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## Concept

The Base $e$ Logarithm Function


$$
y=\ln x \rightarrow e^{y}=x
$$

| Powers of $\boldsymbol{e}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: | :---: |
| $e^{0}=1$ | 1 | 0 |
| $e^{1} \approx 2.718$ | $\approx 2.7$ | 1 |
| $e^{2} \approx 7.4$ | $\approx 7.4$ | 2 |

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## Concept

The logarithmic function of the form $f(x)=a \log _{b}(c(x-h))+k$ has, in general, the following key features.

1. A vertical asymptote at $x=h$.
2. A domain of $x>h$ or $x<h$.
3. A range of all real numbers, $-\infty<y<\infty$.

Objective: Graph Logarithmic Functions and State Key Features
Ex) Graph $g(x)=-3 \log x+2$. Complete the key features. Estimate when necessary.


$$
10^{\prime}=10
$$


transformations $(10,1)$ $\frac{\text { transformations }}{a=-3 \rightarrow \underset{x-a x i s}{v i c t i c h}}$ vert.
$k=2 \mathrm{up}$
Domain: $\{x \mid x>0\}$ set $\}\left(\frac{(0, \infty)}{\text { int. }}\right.$ Range: $\{y \mid-\infty<y<\infty\}(-\infty, \infty)$ Asymptote: $x=0(y$-axis $)$

Zero: $\approx 4$ $\qquad$ $y$-intercept: $\qquad$ none

End Behavior: as $x \rightarrow 0^{+}, g(x) \rightarrow \infty /$ as $x \rightarrow \infty, g(x) \rightarrow-\infty$ Increasing function Decreasing function

Objective: Graph Logarithmic Functions and State Key Features
Ex) Graph $g(x)=0.5 \ln (x+2)$. Complete the key features. Estimate when necessary.

base $=e$ powers of $e$

$$
\begin{array}{ll}
e^{0}=1 & (\text { pour exp. } \\
e^{1} \approx 2.7 & (1,0) \\
e^{2} \approx 7.4 & (2.7,1) \\
\vec{x} & \\
& (7.4,2)
\end{array}
$$

transformations $a=0.5 \rightarrow$ vert. comp. $h=-2$ left 2

Domain: $\{x \mid x>-2\},(-2, \infty)$ Range: $\{y$ $\{y \mid-\infty<y<00\},(-\infty, \infty)$

Asymptote: $\qquad$ $x=-2$

Zero: $\qquad$ $-1$ $y$-intercept: $(0, \approx 0.4)$
End Behavior: as $\rightarrow-2^{+}, g(x) \rightarrow-\infty$, as $x \rightarrow \infty, g(x) \rightarrow \infty$ Increasing function (Decreasing function

Objective: Graph Logarithmic Functions and State Key Features
Ex) Graph $g(x)=\log _{2}(4-x)$. Complete the key features. Estimate when necessary.

base $=2$
powers of 2

| $2^{-1}=\frac{1}{2}$ | pour exp. |
| :--- | :--- |
| $2^{0}=1$ | $\left(\frac{1}{2},-1\right)$ |
| $2^{1}=2$ | $(1,0)$ |
| $2^{2}=4$ | $(2,1)$ |
| $2^{3}=8$ | $(4,2)$ |
| transformations | $(8,3)$ |
| $n=4 \rightarrow$ right | $(8$-axis refl. |
| $\frac{1}{b}=-1 \rightarrow y$ - |  |

Domain:

$$
\{\{x \mid x<4\},(-\infty, 4)
$$ Range: $\{y \mid-\infty<y<\infty\} /(-\infty, \infty)$

Asymptote: $\qquad$ $x=4$

Zero: 3 $\qquad$ $y$-intercept: $\qquad$ $(0,2)$

End Behavior: $\operatorname{as} x \rightarrow-\infty, g(x) \rightarrow \infty$, as $x \rightarrow 4^{-}, g(x) \rightarrow-\infty$ Increasing function Decreasing function (D) (8) (B) (9)

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## Closure

Will a logarithmic function always, sometimes, or never have a $y$-intercept? Explain.

A logarithmic function will sometimes have a $y$-intercept. There will be a $y$-intercept if 0 is part of the domain.

Will a logarithmic function always, sometimes, or never have a zero? Explain.

A logarithmic function will always have a zero because the range of a logarithmic function is the set of all real numbers, which includes the $y$ value of 0 .

