Objective: Graph rational functions using transformations.

asymptote: a line which a function approaches as the value of x goes to positive infinity, negative infinity, or a specific value.

Given a rational function of the form $f(x) = a\left(\dfrac{1}{\frac{1}{b}(x-h)}\right)+k$,

the vertical asymptote has the equation $x = h$

and

the horizontal asymptote has the equation $y = k$.

To graph a simple rational function:

1. Find and graph the vertical and horizontal asymptotes.
2. Find a point on the left side of the vertical asymptote and a point on the right side of the vertical asymptote.
3. Graph the pieces of the curve. Make sure each piece of the curve approaches the asymptotes.

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Ex) a) Find the equations of the vertical and horizontal asymptotes.
b) Graph the function. Include dashed lines for the asymptotes.
c) Find the domain and range of the function as an inequality and as an interval.

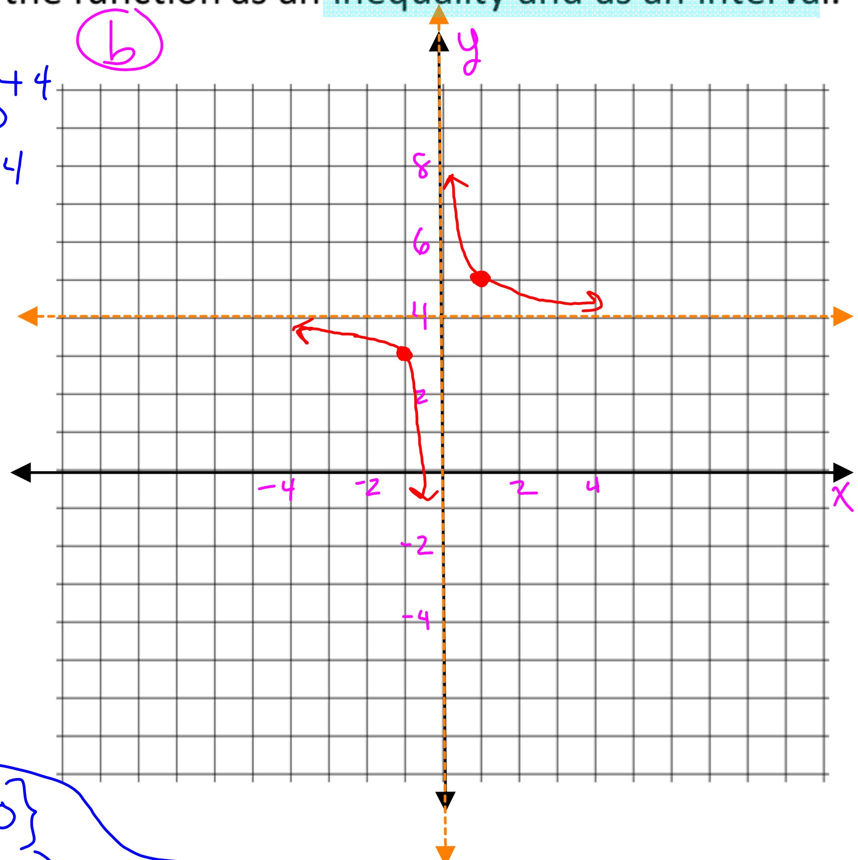
$$g(x) = \frac{1}{x} + 4 \rightarrow g(x) = \frac{1}{x-0} + 4$$

$h = 0,\ k = 4$

a) vert. $x = 0$
horiz. $y = 4$

b) let $x = 1$ $(1, 5)$
$g(1) = \frac{1}{1} + 4 = 5$
let $x = -1$ $(-1, 3)$
$g(-1) = \frac{1}{-1} + 4 = 3$

b)



c) domain
ineq. $\{x \mid x < 0 \text{ or } x > 0\}$
$(-\infty, 0) \cup (0, +\infty)$
range ineq. $\{y \mid y < 4 \text{ or } y > 4\}$
$(-\infty, 4) \cup (4, +\infty)$

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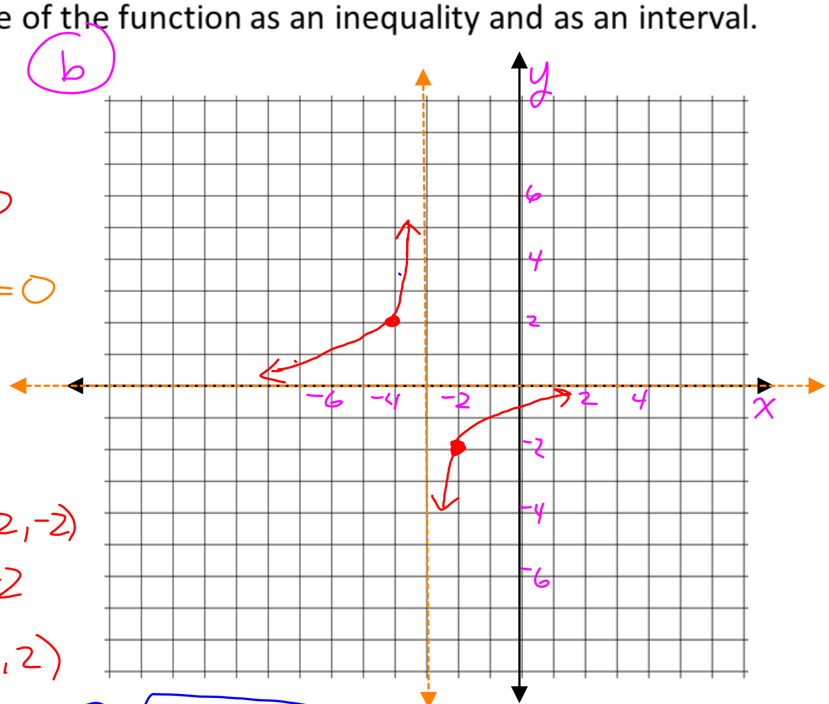
$$h(x) = \frac{-2}{x+3}$$

$$h(x) = -2\left(\frac{1}{x+3}\right) + 0$$

$h = -3 \quad k = 0$

(a) $\boxed{\begin{array}{l} \text{vert. } x = -3 \\ \text{horiz. } y = 0 \end{array}}$

(b)



(b) let $x = -2$ $\quad (-2, -2)$

$h(-2) = \frac{-2}{-2+3} = -2$

let $x = -4$ $\quad (-4, 2)$

$h(-4) = \frac{-2}{-4+3} = 2$

(c) domain

ineq. $\{x \mid x < -3 \text{ or } x > -3\}$

inter. $(-\infty, -3) \cup (-3, +\infty)$

range

ineq. $\{y \mid y < 0 \text{ or } y > 0\}$

interval $(-\infty, 0) \cup (0, +\infty)$

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Ex) a) Find the equations of the vertical and horizontal asymptotes.

b) Graph the function. Include dashed lines for the asymptotes.

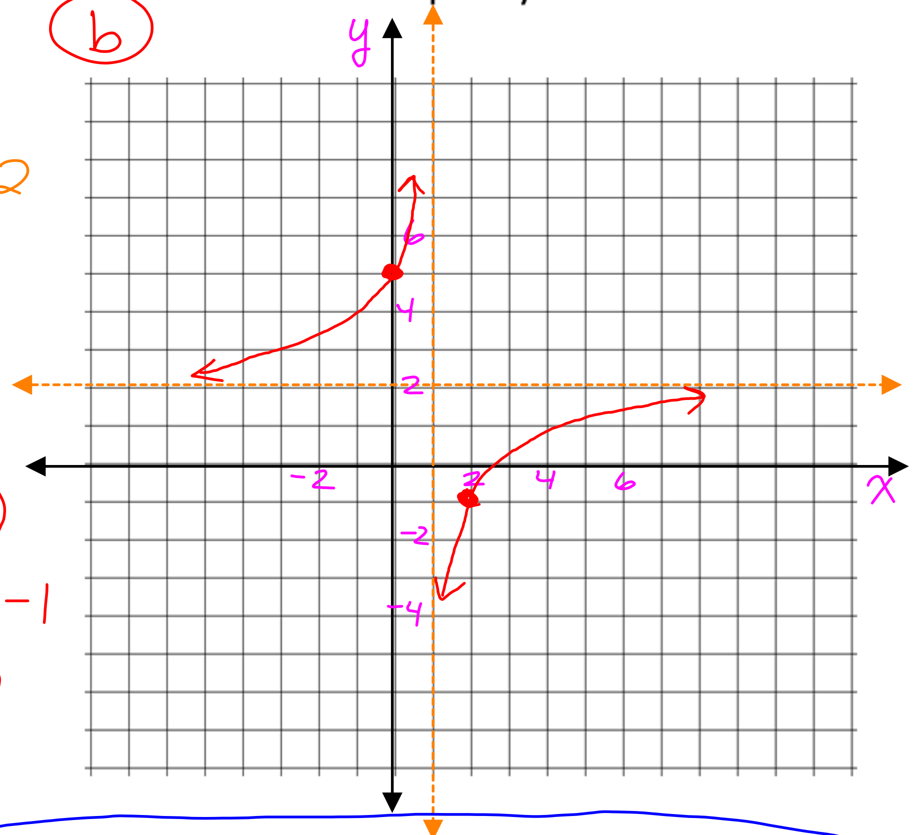
c) Find the domain and range of the function as an inequality and as an interval.

$$d(x) = -3\left(\frac{1}{x-1}\right) + 2$$

$h = 1 \quad k = 2$

a) vertical $x = 1$
horiz. $y = 2$

b



b) let $x = 2 \qquad (2, -1)$

$d(2) = -3\left(\frac{1}{2-1}\right) + 2 = -1$

let $x = 0 \qquad (0, 5)$

$d(0) = -3\left(\frac{1}{0-1}\right) + 2 = 5$

c) domain
ineq. $\{x \mid x < 1 \text{ or } x > 1\}$
interval $(-\infty, 1) \cup (1, +\infty)$

range
ineq. $\{y \mid y < 2 \text{ or } y > 2\}$
interval $(-\infty, 2) \cup (2, +\infty)$

Objective: Graph rational functions using transformations.

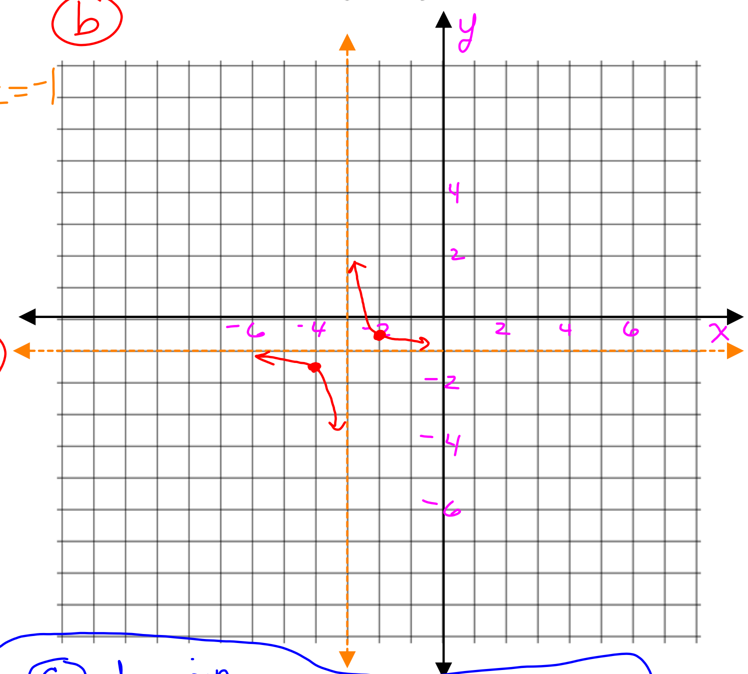
Ex) a) Find the equations of the vertical and horizontal asymptotes.
b) Graph the function. Include dashed lines for the asymptotes.
c) Find the domain and range of the function as an inequality and as an interval.

$$r(x) = \frac{1}{2(x+3)} - 1$$

$h = -3 \quad k = -1$

a) vertical $x = -3$
horiz. $y = -1$

b)



b) let $x = -2$ $\left(-2, -\frac{1}{2}\right)$

$$r(-2) = \frac{1}{2(-2+3)} - 1$$
$$= \frac{1}{2} + -1 = -\frac{1}{2}$$

let $x = -4$ $\left(-4, -1\frac{1}{2}\right)$

$$r(-4) = \frac{1}{2(-4+3)} - 1$$
$$= -\frac{1}{2} + -1 = -1\frac{1}{2}$$

c) domain

ineq. $\{x \mid x < -3 \text{ or } x > -3\}$

interval $(-\infty, -3) \cup (-3, +\infty)$

range

ineq. $\{y \mid y < -1 \text{ or } y > -1\}$

interval $(-\infty, -1) \cup (-1, +\infty)$

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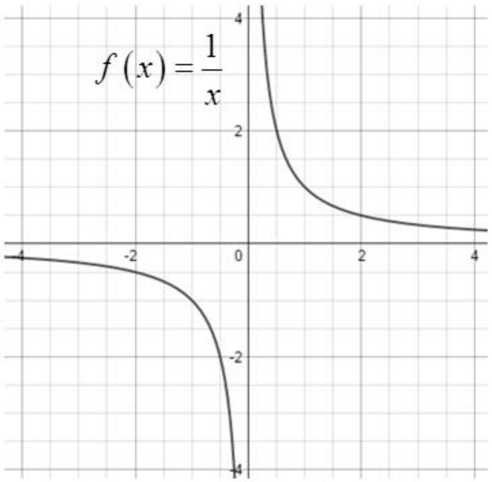
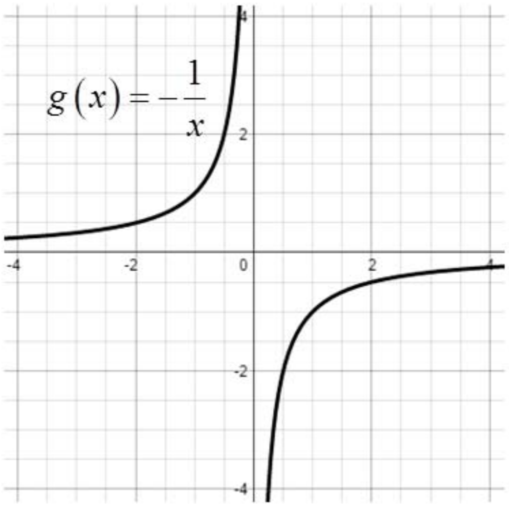
Closure Compare and contrast the characteristics of the graphs of the rational functions

$$f(x) = \frac{1}{x} \text{ and } g(x) = -\frac{1}{x}$$





Both functions have a vertical asymptote at x = 0 and a horizontal asymptote at y = 0. The g(x) function differs from f(x) because it has a horizontal reflection, so f(x) is always decreasing and g(x) is always increasing. Also, the end behavior for the two functions is the same as *x* approaches $+\infty$ and $-\infty$ and different as x approaches 0.