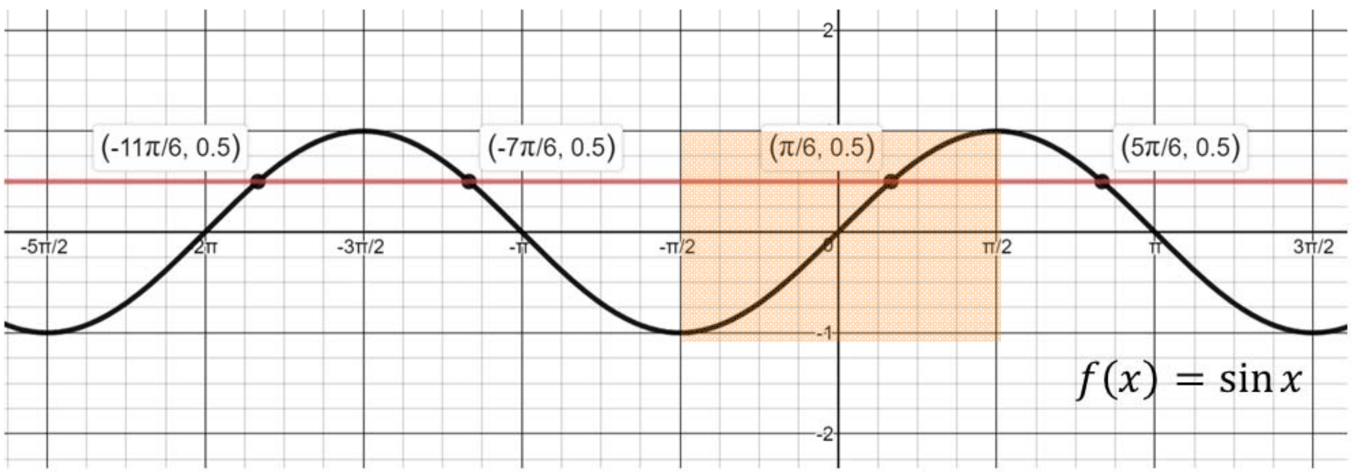
# Objective: Evaluate Inverse Trigonometric Functions

## Concept

In order for a function to have an inverse function, it must pass the Horizontal Line Test. The Horizontal Line Test shows whether each $y$-value corresponds to one $x$-value, a condition necessary for the inverse to be a function.

The sine function does not pass the Horizontal Line Test because the horizontal line intersects the graph at more than one point. So, in order to create an inverse of sine that is a function, the domain of sine must be restricted.



# Objective: Evaluate Inverse Trigonometric Functions

## Concept

**When the domain of a trigonometric function is restricted, the function is capitalized.**

$$\text{Given } f(x) = \sin x\text{; the domain is } -\infty < x < +\infty$$

$$\mathbf{Given}\ \boldsymbol{f(x) = Sin\ x}\mathbf{; the\ domain\ is}\ -\frac{\boldsymbol{\pi}}{\mathbf{2}} \le \boldsymbol{x} \le \frac{\boldsymbol{\pi}}{\mathbf{2}}$$

$$\text{Given } f(x) = \cos x\text{; the domain is } -\infty < x < +\infty$$

$$\mathbf{Given}\ \boldsymbol{f(x) = Cos\ x}\mathbf{; the\ domain\ is\ 0} \le \boldsymbol{x} \le \boldsymbol{\pi}$$

$$\text{Given } f(x) = \tan x\text{; the domain is all } x \ne \frac{\pi}{2} + \pi k, k\ any\ integer$$

$$\mathbf{Given}\ \boldsymbol{f(x) = Tan\ x}\mathbf{; the\ domain\ is}\ -\frac{\boldsymbol{\pi}}{\mathbf{2}} < \boldsymbol{x} < \frac{\boldsymbol{\pi}}{\mathbf{2}}$$

These domain restrictions were chosen so there would be only one angle that corresponds to each possible value of sine, cosine, or tangent.

## Objective: Evaluate Inverse Trigonometric Functions

### Concept

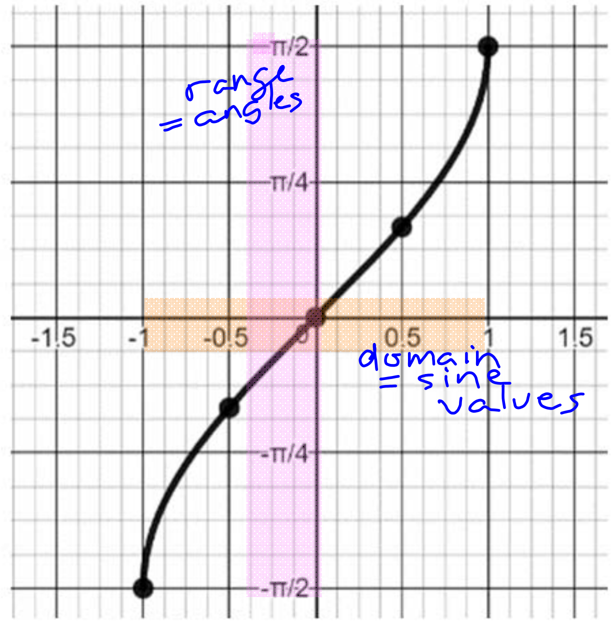
**The Inverse Sine Function**

$$f(x) = \sin^{-1} x \text{ or}$$
$$f(x) = \arcsin x$$

The angles are the $\underline{\boldsymbol{y \text{ or } f(x)}}$ values.

Domain: $\underline{\boldsymbol{-1 \le x \le 1}}$

Range: $\underline{\boldsymbol{-\frac{\pi}{2} \le y \le \frac{\pi}{2}}}$




Note: The *arcsin* notation comes from the fact that in the unit circle the length of the arc subtended by the angle equals the radian measure of that angle. Therefore, the $\arcsin x$ notation means the arc (or angle) that has a sine ratio of $x$.

Objective: Evaluate Inverse Trigonometric Functions

Concept

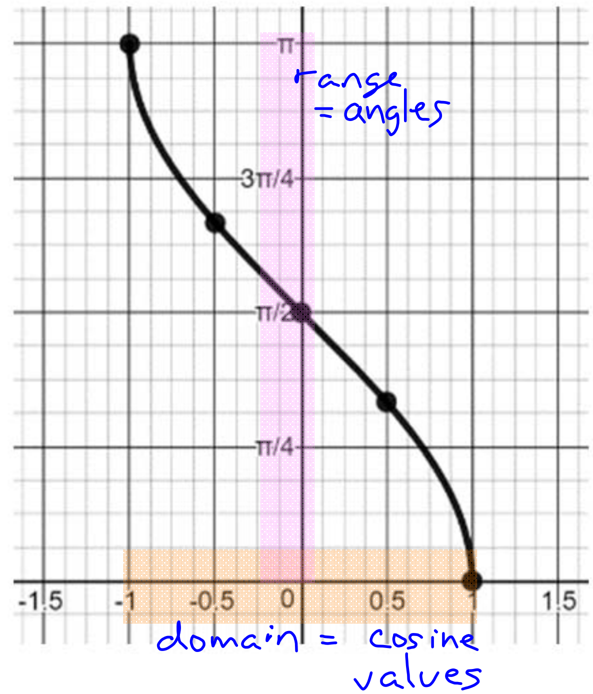
**The Inverse Cosine Function**

$f(x) = \cos^{-1} x$ or
$f(x) = \arccos x$

The angles are the $\boldsymbol{y\ or\ f(x)}$ values.

Domain: $\boldsymbol{-1 \leq x \leq 1}$

Range: $\boldsymbol{0 \leq y \leq \pi}$



Objective: Evaluate Inverse Trigonometric Functions

Concept

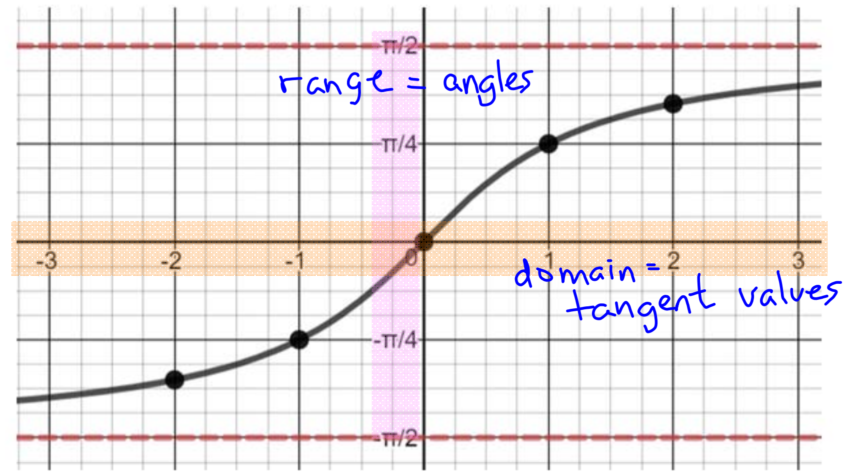
**The Inverse Tangent Function**

$f(x) = \tan^{-1} x$ or
$f(x) = \arctan x$

The angles are the $\boldsymbol{y \text{ or } f(x)}$ values.

Domain: $-\infty < x < +\infty$

Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$



## Objective: Evaluate Inverse Trigonometric Functions

Ex) If possible, find the exact value of each expression.

$\arcsin\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$

$= \sin^{-1}\left(-\frac{1}{2}\right)$

means: what angle has a sine value of $\frac{-1}{2}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

$= \arccos\left(\frac{-\sqrt{2}}{2}\right)$

means: what angle has a cosine value of $-\frac{\sqrt{2}}{2}$ in $[0, \pi]$

$\sin^{-1}(2) = \boxed{\text{not possible}}$

$= \arcsin(2)$

means: what angle has a sine value of 2?

## Objective: Evaluate Inverse Trigonometric Functions

Ex) If possible, find the exact value of each expression.

$\tan^{-1}(1) = \boxed{\frac{\pi}{4}}$

$= \arctan(1)$

means: what angle has a tangent value of 1 in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$arctan\left(-\frac{\sqrt{3}}{3}\right) = \boxed{-\frac{\pi}{6}}$

$= \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

means: what angle has a tangent value of $-\frac{\sqrt{3}}{3}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

## Objective: Evaluate Inverse Trigonometric Functions

Ex) If possible, find the exact value of each expression.

$$\arcsin\left(\sin\frac{3\pi}{2}\right) = \boxed{-\frac{\pi}{2}}$$

1

$= \arcsin(-1)$

angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

$= \boxed{-\frac{\pi}{2}}$

$$\cos(\cos^{-1}(-1)) = \boxed{-1}$$

1

angle in $[0, \pi]$?

$= \cos(\pi)$

$= \boxed{-1}$

## Objective: Evaluate Inverse Trigonometric Functions

Ex) If possible, find the exact value of each expression.

$$\arcsin\left(\cos\frac{\pi}{3}\right) = \boxed{\frac{\pi}{6}}$$

$$= \arcsin\left(\frac{1}{2}\right)$$

angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

$$= \boxed{\frac{\pi}{6}}$$

$$\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \boxed{\frac{-\sqrt{3}}{3}}$$

angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$= \tan\left(\frac{-\pi}{6}\right)$$

$$= \frac{\sin\frac{-\pi}{6}}{\cos{-\pi/6}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

## Objective: Evaluate Inverse Trigonometric Functions

### Closure

Match each range interval to the correct inverse trigonometric function.

**Range**

1. $f(x) = arcsin\,x$ A. $[0, \pi]$

2. $f(x) = arccos\,x$ B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. $f(x) = arctan\,x$ C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$