Objective: Simplify Quotients of Radical Expressions

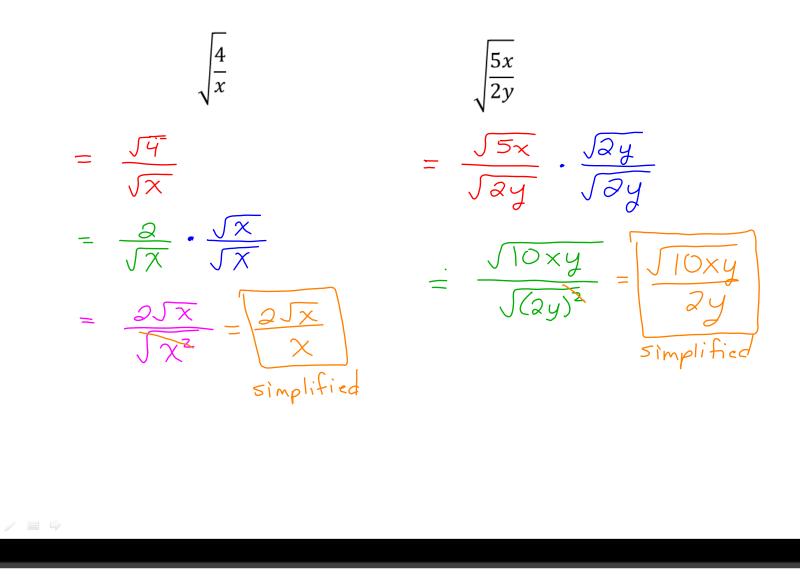### Concept

Rationalizing a Denominator

For an expression with a denominator to be in simplest form, **the denominator must be an expression that doesn't involve any radical expression in any form** (no roots or fractional exponents).

The math that must be done to complete this process involves multiplying the expression by a ratio of 1. The form the ratio of 1 must take differs for different types of expressions.
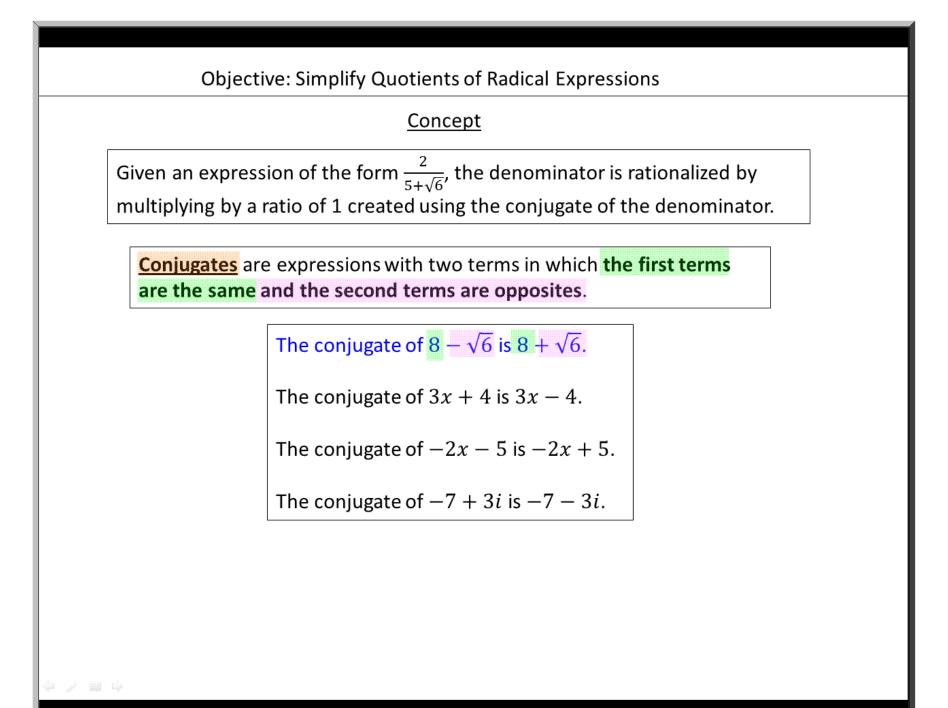
$$not\ simplified \quad \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \boxed{\frac{\sqrt{3}}{3}}\ simplified$$

Objective: Simplify Quotients of Radical Expressions

Ex) Simplify each expression. Assume all variables are positive. Simplify numerical values as much as possible.

$$\sqrt{\frac{4}{x}}$$

$$= \frac{\sqrt{4}}{\sqrt{x}}$$

$$= \frac{2}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{2\sqrt{x}}{\sqrt{x^2}} = \boxed{\frac{2\sqrt{x}}{x}}$$

simplified

$$\sqrt{\frac{5x}{2y}}$$

$$= \frac{\sqrt{5x}}{\sqrt{2y}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}}$$

$$= \frac{\sqrt{10xy}}{\sqrt{(2y)^2}} = \boxed{\frac{\sqrt{10xy}}{2y}}$$
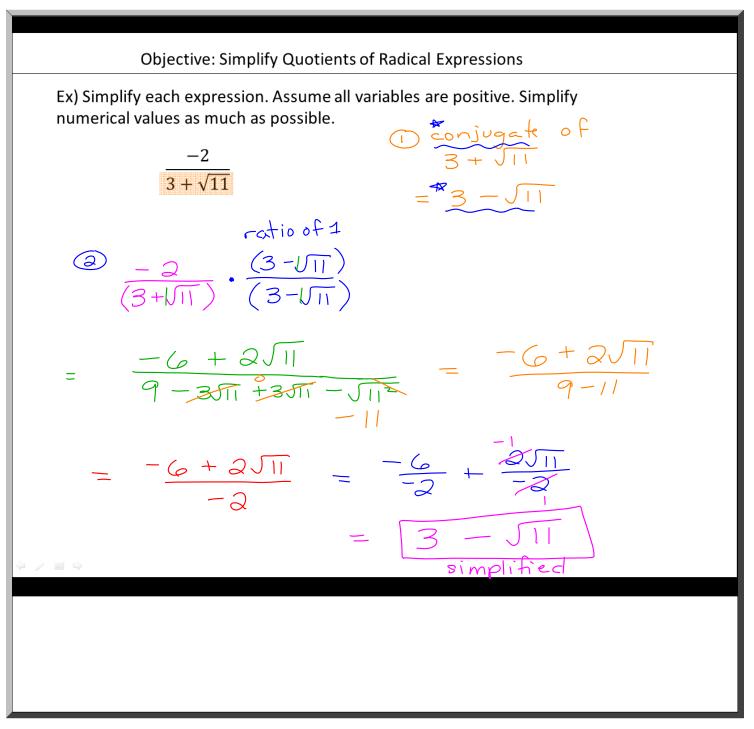
simplified

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Ex) Simplify each expression. Assume all variables are positive. Simplify numerical values as much as possible.

$$\sqrt[3]{\dfrac{27}{x}} \qquad\qquad \sqrt[3]{\dfrac{16}{x^2}}$$

$$= \dfrac{\sqrt[3]{27}}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

$$= \dfrac{3\sqrt[3]{x^2}}{\sqrt[3]{x^3}}$$

$$= \boxed{\dfrac{3\sqrt[3]{x^2}}{x}}$$

simplified

$$= \dfrac{\sqrt[3]{16}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{8}\cdot\sqrt[3]{2}}{\sqrt[3]{x^2}}$$

$$= \dfrac{2\sqrt[3]{2}}{\sqrt[3]{x^2}} \cdot \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}}$$

$$= \dfrac{2\sqrt[3]{2x}}{\sqrt[3]{x^3}}$$

$$= \boxed{\dfrac{2\sqrt[3]{2x}}{x}}$$

simplified

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Concept

Given an expression of the form $\dfrac{2}{5+\sqrt{6}}$, the denominator is rationalized by multiplying by a ratio of 1 created using the conjugate of the denominator.

Conjugates are expressions with two terms in which the first terms are the same and the second terms are opposites.

The conjugate of $8 - \sqrt{6}$ is $8 + \sqrt{6}$.

The conjugate of $3x + 4$ is $3x - 4$.

The conjugate of $-2x - 5$ is $-2x + 5$.

The conjugate of $-7 + 3i$ is $-7 - 3i$.

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Ex) Simplify each expression. Assume all variables are positive. Simplify numerical values as much as possible.

$$\frac{-2}{3 + \sqrt{11}}$$

① conjugate of $\dfrac{}{3 + \sqrt{11}}$

$= 3 - \sqrt{11}$

ratio of 1

② $\dfrac{-2}{(3 + \sqrt{11})} \cdot \dfrac{(3 - \sqrt{11})}{(3 - \sqrt{11})}$

$= \dfrac{-6 + 2\sqrt{11}}{9 - 3\sqrt{11} + 3\sqrt{11} - \sqrt{11}^2}$ $= \dfrac{-6 + 2\sqrt{11}}{9 - 11}$

-11

$= \dfrac{-6 + 2\sqrt{11}}{-2}$ $= \dfrac{-6}{-2} + \dfrac{2\sqrt{11}}{-2}$

$= \boxed{3 - \sqrt{11}}$

simplified

Captured on Fri Nov 03 2017 08:22:15

Objective: Simplify Quotients of Radical Expressions

Ex) Simplify each expression. Assume all variables are positive. Simplify numerical values as much as possible.

$$\frac{10}{-6 - \sqrt{2}}$$

① conjugate of $-6 - \sqrt{2}$

$$= -6 + \sqrt{2}$$

② $\dfrac{10}{(-6 - \sqrt{2})} \cdot \dfrac{(-6 + \sqrt{2})}{(-6 + \sqrt{2})} = \dfrac{-60 + 10\sqrt{2}}{36 - 6\sqrt{2} + 6\sqrt{2} - \sqrt{2}^2}$$

$$-2$$

$$= \frac{-60 + 10\sqrt{2}}{36 - 2} = \frac{-60 + 10\sqrt{2}}{34}$$

$$= \frac{-60}{34} + \frac{10\sqrt{2}}{34} = \boxed{\frac{-30}{17} + \frac{5\sqrt{2}}{17}}$$

simplified

Objective: Simplify Quotients of Radical Expressions

Ex) Simplify each expression. Assume all variables are positive. Simplify numerical values as much as possible.

$$\frac{2 - \sqrt{5}}{5 - \sqrt{5}}$$

① conjugate of $5 - \sqrt{5}$

$$= 5 + \sqrt{5}$$

② $\dfrac{(2 - \sqrt{5})}{(5 - \sqrt{5})} \cdot \dfrac{(5 + \sqrt{5})}{(5 + \sqrt{5})} = \dfrac{10 + 2\sqrt{5} - 5\sqrt{5} - \sqrt{5}^2}{25 + 5\sqrt{5} - 5\sqrt{5} - \sqrt{5}^2}$$

$$= \frac{5 - 3\sqrt{5}}{20} = \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} - \frac{3\sqrt{5}}{20} = \boxed{\frac{1}{4} - \frac{3\sqrt{5}}{20}}$$

simplified

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Closure

Hannah is going to simplify the radical expression $\dfrac{-5}{-2+\sqrt{15}}$. Which ratio of 1 should she use? Explain your reasoning.

A) $\dfrac{-2+\sqrt{15}}{-2+\sqrt{15}}$ B) $\dfrac{2-\sqrt{15}}{2-\sqrt{15}}$ C) $\dfrac{-2-\sqrt{15}}{-2-\sqrt{15}}$ D) $\dfrac{2+\sqrt{15}}{2+\sqrt{15}}$

Hannah should use choice C, $\dfrac{-2-\sqrt{15}}{-2-\sqrt{15}}$, because this ratio of 1 is created using the conjugate of the denominator.