Objective: Solve cube root equations algebraically.

Concept

A Radical Equation contains a variable within a radical or a variable raised to a non-integer rational exponent.

Examples

$\sqrt{x-4} = 7$ $\quad (x+5)^{\frac{1}{2}} = 9$ $\quad \sqrt[3]{x^2-5} = 2$

Non-Examples

$\sqrt{x} + 9$ (no equal sign)

$x - 12 = 6$ (no radical or non-integer rational exponent)

$(x+4)^{\frac{1}{2}}$ (no equal sign)

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Concept

Steps to Solve a Radical Equation

1. **Isolate the radical expression.** If the equation contains more than one radical expression, choose one to isolate.
2. **Raise both sides of the equation to the appropriate power** so the isolated root and power cancel.
3. **Solve the resulting equation.** Be aware of whether the equation is linear or quadratic.
4. **Check for Extraneous Solutions** and then write the final solution set.

Radical equations can have **extraneous solutions**:

1. Solutions that fail to make the left side and right side of the equation equal.
2. Solutions that are imaginary or create imaginary values when substituted into the original equation.

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Ex) Solve the equation.

① $\sqrt[3]{x} = 2$

② cube both sides $\left(\sqrt[3]{x}\right)^3 = (2)^3$

③ $x = 8$

④ check. $\sqrt[3]{8} = 2$

$2 = 2$ ✓

solution: $x = 8$

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Ex) Solve the equation.

(1) $\sqrt[3]{x} = -4$

(2) cube both sides

$\left(\sqrt[3]{x}\right)^3 = (-4)^3$

$-4 \cdot -4 \cdot -4$

$16 \cdot -4$

(3) $x = -64$

(4) check.

$\sqrt[3]{-64} \stackrel{?}{=} -4$

$-4 = -4$ ✓

solution: $x = -64$

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Ex) Solve the equation.

$$5\sqrt[3]{3x+1} = -10$$

① $\frac{5 \cdot \sqrt[3]{3x+1}}{5} = \frac{-10}{5}$

$\sqrt[3]{3x+1} = -2$

② cube both sides $\left(\sqrt[3]{3x+1}\right)^3 = (-2)^3$

$-2 \cdot -2 \cdot -2$

③ $3x + 1 = -8$

$-1 \quad -1$

$\frac{3x}{3} = \frac{-9}{3}$

$x = -3$

④ check. $5 \cdot \sqrt[3]{3(-3)+1} \stackrel{?}{=} -10$

$-9+1$

$5 \cdot \sqrt[3]{-8}$

$5 \cdot -2 = -10$

$-10 = -10$ ✓

Solution: $x = -3$

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Practice) Solve the equation.

$$\sqrt[3]{5-x}+7=6$$

$$\sqrt[3]{5-x}+7=6$$

$$\quad -7 \quad -7$$

$$\sqrt[3]{5-x}=-1$$

$$\left(\sqrt[3]{5-x}\right)^3=(-1)^3$$

$$5-x=-1$$

$$+1 \quad +x \quad +1+x$$

$$6=x$$

$$check: x=6;\ \sqrt[3]{5-6}+7=6 \to \sqrt[3]{-1}+7=6 \to -1+7=6(yes)$$

$$\boxed{solution: x=6}$$

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Ex) Solve the equation.

$$8-(2x+5)^{\frac{1}{3}}=3$$

same as: $8-\sqrt[3]{2x+5}=3$

① $-3+\sqrt[3]{2x+5} \quad -3+\sqrt[3]{2x+5}$

$5=\sqrt[3]{2x+5}$

② $(5)^3=\left(\sqrt[3]{2x+5}\right)^3$

$5\cdot 5\cdot 5$

③ $125=2x+5$

$-5 \quad -5$

$\frac{120}{2}=\frac{2x}{2}$

$x=60$

④ check.

$8-\sqrt[3]{2(60)+5}\overset{?}{=}3$

$120+5$

$8-\sqrt[3]{125}$

$8-5=3$ ✓

solution: $x=60$

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Practice) Solve the equation.

$$(5x-3)^{\frac{1}{3}}-4=-1$$

$$\sqrt[3]{5x-3}-4=-1$$

$$+4 \quad +4$$

$$\sqrt[3]{5x-3}=3$$

$$\left(\sqrt[3]{5x-3}\right)^3=(3)^3$$

$$5x-3=27$$

$$+3 \quad +3$$

$$5x=30$$

$$x=6$$

$$check: x=6;\ \sqrt[3]{5(6)-3}-4=-1 \rightarrow \sqrt[3]{27}-4=-1 \rightarrow 3-4=-1(yes)$$

$$\boxed{solution: x=6}$$

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Practice) Solve the equation.

$$2(x-50)^{\frac{1}{3}} = -10$$

$$\frac{2\sqrt[3]{x-50}}{2} = \frac{-10}{2}$$

$$\sqrt[3]{x-50} = -5$$

$$\left(\sqrt[3]{x-50}\right)^3 = (-5)^3$$

$$x-50 = -125$$

$$+50 \quad +50$$

$$x = -75$$

$$check: x = -75;\ 2\cdot\sqrt[3]{-75-50} = -10 \rightarrow 2\cdot\sqrt[3]{-125} = -10 \rightarrow 2\cdot -5 = -10\ (yes)$$

$$\boxed{solution: x = -75}$$

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Ex) Solve the equation.

(1) $\sqrt[3]{x^2+9} = 2$

(2) $\left(\sqrt[3]{x^2+9}\right)^3 = (2)^3$

(3) $x^2 + 9 = 8$

$\quad -9 \quad -9$

$x^2 = -1$

$\sqrt{x^2} = \pm\sqrt{-1}$

$x = \pm i, i$ imaginary

extraneous

(4) no solution or $\emptyset$

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Ex) Solve the equation.

① $\sqrt[3]{x^2 - 7} - 1 = 6$

$+1 \quad +1$

$\sqrt[3]{x^2-7} = 7$

② $\left(\sqrt[3]{x^2-7}\right)^3 = (7)^3$

$49 \times 7 = 343$

③ $x^2 - 7 = 343$

$+7 \quad +7$

$x^2 = 350$

$\sqrt{x^2} = \pm\sqrt{350}$

$\sqrt{25}\cdot\sqrt{14}$

$x = \pm 5\sqrt{14}$

④ check.

$\sqrt[3]{(5\sqrt{14})^2 - 7} - 1 \overset{?}{=} 6$

$25 \cdot 14$

solutions

$x = -5\sqrt{14}, 5\sqrt{14}$

$\sqrt[3]{350-7} - 1$

$\sqrt[3]{343} - 1$

$7 - 1 = 6$ ✓

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Practice) Solve the equation.

$$\sqrt[3]{x^2+2}=3$$

$$\sqrt[3]{x^2+2}=3$$

$$\left(\sqrt[3]{x^2+2}\right)^3=(3)^3$$

$$x^2+2=27$$

$$\underline{\quad -2 \quad -2}$$

$$x^2=25$$

$$\sqrt{x^2}=\pm\sqrt{25}$$

$$x=-5,\ 5$$

$check:\ x=-5;\ \sqrt[3]{(-5)^2+2}=3\rightarrow\sqrt[3]{27}=3\ (yes)$

$x=5;\ \sqrt[3]{(5)^2+2}=3\rightarrow\sqrt[3]{27}=3\ (yes)$

$\boxed{solution:\ x=-5,\ 5}$

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Closure

Which of the following types of solutions will always be extraneous when solving a radical equation? Choose all that apply.

A) Rational Numbers
B) Irrational Numbers
C) Imaginary Numbers