Objective: Solve Exponential Equations Using Logarithms

Concept

An **exponential equation** is an equation where **the variable is in the exponent expression**.

Examples

$$5^x = 6$$

$$2^{3x-5} = 4^{5x}$$

$$8^{0.2t} = 7.6^{t+2}$$

Non-examples

$$5^3 = x + 8$$

(the variable is not the exponent)

$$x^2 = 36$$

(the variable is not the exponent)

Objective: Solve Exponential Equations Using Logarithms

Concept

If an exponential equation cannot be solved by writing the powers with the same base, sometimes the equation can be rewritten in logarithmic form and solved for the variable. Other times the Property of Equality for Logarithms must be used.

Steps to Solve an Exponential Equation Using Logarithms

1. Write the equation with a single term on each side.
2. Take the logarithm of both sides. (Common Logarithm or Natural Logarithm)
3. Use the Power Property of Logarithms: $\log_b a^x = x log_b a$
4. Solve this equation using algebra. Approximate if necessary.
5. State the solution to the exponential equation.

Property of Equality for Logarithms

For positive numbers $x, y, and\ b\ (b \neq 1)$

If $\quad x = y$

Then $\quad log_b x = log_b y$

Objective: Solve Exponential Equations Using Logarithms

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$5^x - 4 = 7$$

1 $\quad +4 \quad +4$

$$5^x = 11$$

with common log

2 $\log 5^x = \log 11$

3 $x \cdot \log 5 = \log 11$

4 $\dfrac{x \cdot \cancel{\log 5}}{\cancel{\log 5}} = \dfrac{\log 11}{\log 5}$

5 $\boxed{x = \dfrac{\log(11)}{\log(5)} \quad x \approx 1.490}$

"ln" with natural log

2 $\ln 5^x = \ln 11$

3 $x \cdot \ln 5 = \ln 11$

4 $\dfrac{x \cdot \cancel{\ln 5}}{\cancel{\ln 5}} = \dfrac{\ln 11}{\ln 5}$

5 $\boxed{x = \dfrac{\ln(11)}{\ln(5)} \quad x \approx 1.490}$

Objective: Solve Exponential Equations Using Logarithms

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

$$\frac{2(7)^{3x+2}}{2} = \frac{500}{2}$$

1

$$7^{3x+2} = 250$$

2

$$\ln 7^{3x+2} = \ln 250$$

3

$$(3x+2)\cdot \ln 7 = \ln 250$$

4

$$3x\cdot\ln 7 + 2\cdot\ln 7 = \ln 250$$

$$-2\ln 7 \qquad -2\ln 7$$

$$\frac{3x\cdot\ln 7}{(3\cdot\ln 7)} = \frac{\ln 250 - 2\ln 7}{(3\cdot\ln 7)}$$

5

$$\boxed{x = \frac{(\ln(250) - 2\ln(7))}{(3\cdot\ln(7))} \qquad x \approx 0.279}$$

Objective: Solve Exponential Equations Using Logarithms

Ex) Solve the equation. Give the exact solution and the approximate solution to three decimal places.

1. $3^{x-5} = 13^{x}$

2. $\ln 3^{x-5} = \ln 13^{x}$

3. $(x-5)\cdot\ln 3 = x\cdot\ln 13$

4. $x\cdot\ln 3 - 5\cdot\ln 3 = x\cdot\ln 13$

$-x\cdot\ln 3 \qquad\qquad -x\cdot\ln 3$

$-5\cdot\ln 3 = x\cdot\ln 13 - x\cdot\ln 3$

gcf

$$\frac{-5\ln 3}{(\ln 13 - \ln 3)} = \frac{x\,\cancel{(\ln 13 - \ln 3)}}{\cancel{(\ln 13 - \ln 3)}}$$

5. $$\boxed{x = \frac{-5\ln(3)}{(\ln(13) - \ln(3))} \qquad x \approx -3.746}$$

Objective: Solve Exponential Equations Using Logarithms

Closure

Jeff solved the equation shown. Explain his errors in steps 1 and 2 and then find the correct solution.

$$4^{x+2} = 9$$

$$\begin{aligned} &step\,1: && \ln 4^{x+2} = 9 \\ &step\,2: && x + 2\ln 4 = 9 \\ &step\,3: && \underline{\;-2\ln 4 \quad -2\ln 4\;} \end{aligned}$$

$$\boxed{\begin{aligned} &x = 9 - 2\ln 4 \\ &x \approx 6.227 \end{aligned}}$$

Jeff's error in step 1 is that he only took the logarithm of one side of the equation. His error in step 2 is that he didn't put parentheses around the exponent of $(x+2)$. The correct solution is $x = \frac{\ln 9 - 2\ln 4}{\ln 4}$ or $x \approx -0.415$.