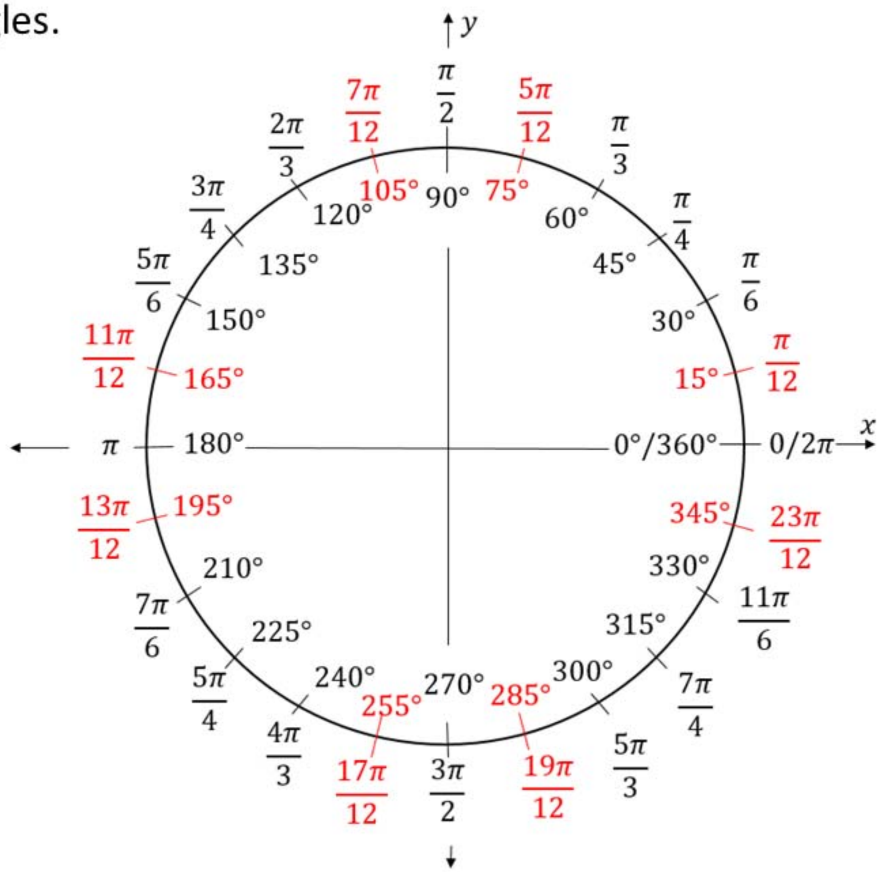
Objective: Use the sum and difference identities to evaluate trigonometric functions

Concept

Exact Values of the trigonometric functions of other angles can sometimes be found using identities. The sum and difference identities allow us to find the exact values of trigonometric functions of angles where the angle value is equal to the sum or difference of two unit circle angles.

$$45° - 30° = 15°$$
$$\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

$$45° + 30° = 75°$$
$$\frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$$



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Concept

*To find an exact trigonometric value for angles with reference angles of $\frac{\pi}{12}$ *and* $\frac{5\pi}{12}$ ($15°$ *and* $75°$) use the sum and difference identities.

Sum and Difference Identities for Trigonometric Functions *
$sin(\alpha+\beta) = sin\,\alpha\cos\beta + \cos\alpha\sin\beta$
$sin(\alpha-\beta) = sin\,\alpha\cos\beta - \cos\alpha\sin\beta$
$cos(\alpha+\beta) = cos\,\alpha\cos\beta - \sin\alpha\sin\beta$
$cos(\alpha-\beta) = cos\,\alpha\cos\beta + \sin\alpha\sin\beta$

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Steps for Using a Sum or Difference Identity

1. Write the angle as a sum or difference of unit circle angles.
2. Use the corresponding identity.
3. Substitute the unit circle angles into the identity for α and β.
4. Substitute the corresponding values of sine and cosine into the identity.
5. Simplify.
6. Write the final answer.

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Ex) Find the exact value of $\cos\frac{\pi}{12}$.

① $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ $(\alpha - \beta)$

② $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

③ $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3} \cdot \cos\frac{\pi}{4} + \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{4}$

④ $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

⑤ $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

⑥ $\boxed{\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2} + \sqrt{6}}{4}}$

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Ex) Find the exact value of $\sin\frac{17\pi}{12}$.

① $\frac{17\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$

$(\alpha + \beta)$

② $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$

③ $\sin\left(\frac{5\pi}{4}+\frac{\pi}{6}\right) = \sin\frac{5\pi}{4}\cdot\cos\frac{\pi}{6} + \cos\frac{5\pi}{4}\cdot\sin\frac{\pi}{6}$

④ $\sin\left(\frac{5\pi}{4}+\frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

⑤ $\sin\left(\frac{17\pi}{12}\right) = \frac{-\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

⑥ $\boxed{\sin\left(\frac{17\pi}{12}\right) = \frac{-\sqrt{6}-\sqrt{2}}{4}}$

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Closure

Cheryl is finding the exact value of $\cos\frac{7\pi}{12}$. Her work is shown.
Identify the two errors Cheryl made.

$$\cos\frac{7\pi}{12}=\cos\left(\frac{3\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{3\pi}{4}\cos\frac{\pi}{6}-\sin\frac{3\pi}{4}\sin\frac{\pi}{6}$$

$$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\boxed{\cos\frac{7\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}}$$

The first error Cheryl made is that the identity should be $\cos\alpha\cos\beta+\sin\alpha\sin\beta$. The second error is that the $\cos\frac{3\pi}{4}$ is negative, not positive.