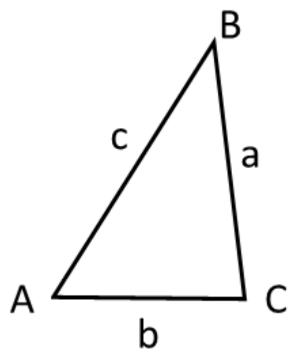
Objective: Solve Non-Right Triangles Using the Law of Sines

Concept

The definitions of sine, cosine, and tangent are based on the ratios of the sides of a <u>right triangle</u>. Since there is no hypotenuse of a non-right triangle, these definitions <u>cannot be used to solve non-right triangles</u>.



For non-right triangle ABC:

$\sin A \neq \frac{opp}{hyp}$ $\cos A \neq \frac{adj}{hyp}$ $\tan A \neq \frac{opp}{adj}$

And, $a^2 + b^2 \neq c^2$.

For non-right triangles, the Law of Sines can sometimes be used to solve the triangle.

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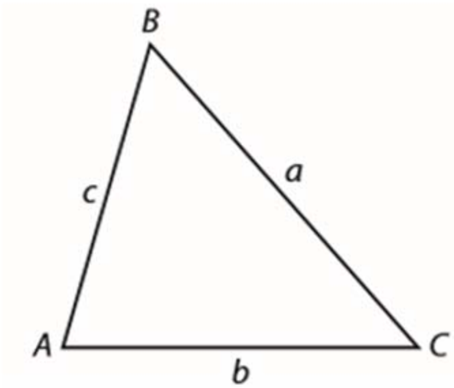

The Law of Sines

Given ΔABC

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Note: When using the Law of Sines, two ratios are used to create a proportion.

Given ΔABC

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Given ΔABC

$$\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Given ΔABC

$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$$

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Concept

Solving a Non-Right Triangle Using the Law of Sines

- **Given two sides measures and one opposite angle measure.**
  1. Use the Law of Sines to find the second opposite angle.
  2. Use the Triangle Sum Theorem to find the third angle.
  3. Use the Law of Sines to find the third side measure.

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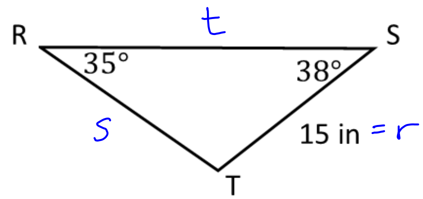
Ex) Solve the triangle. When necessary, round side measures to the nearest tenth and angle measures to the nearest degree.



① find side s

$$\frac{\sin R}{r} = \frac{\sin S}{s}$$

$$\frac{\sin 35^\circ}{15} = \frac{\sin 38^\circ}{s}$$

$$\frac{s \cdot \sin 35^\circ}{\sin 35^\circ} = \frac{15 \cdot \sin 38^\circ}{\sin 35^\circ}$$

$$s = \frac{15 \cdot \sin 38^\circ}{\sin 35^\circ}$$

$$\boxed{s \approx 16.1 \text{ inches}}$$

② find $m\angle T$

$$m\angle R + m\angle S + m\angle T = 180^\circ$$

$$35^\circ + 38^\circ + m\angle T = 180^\circ$$

$$\boxed{m\angle T = 107^\circ}$$

③ find side t

$$\frac{\sin R}{r} = \frac{\sin T}{t}$$

$$\frac{\sin 35^\circ}{15} = \frac{\sin 107^\circ}{t}$$

$$\frac{t \cdot \sin 35^\circ}{\sin 35^\circ} = \frac{15 \cdot \sin 107^\circ}{\sin 35^\circ}$$

$$t = \frac{15 \cdot \sin 107^\circ}{\sin 35^\circ}$$

$$\boxed{t \approx 25.0 \text{ inches}}$$

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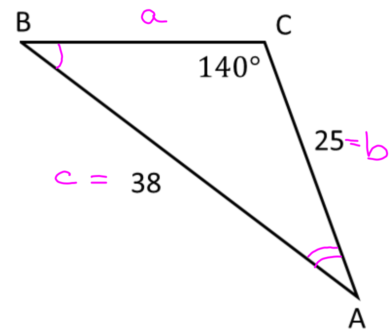

① find $m\angle B$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 140^\circ}{38} = \frac{\sin B}{25}$$

$$\frac{25 \cdot \sin 140^\circ}{38} = \frac{38 \sin B}{38}$$

$$\sin B = \frac{25 \cdot \sin 140^\circ}{38}$$

$$\cancel{\sin^{-1}}(\cancel{\sin} B) = \sin^{-1}\left(\frac{25 \cdot \sin 140^\circ}{38}\right)$$

$$m\angle B = \sin^{-1}\left(\frac{25 \cdot \sin 140^\circ}{38}\right)$$

$$\boxed{m\angle B \approx 25^\circ}$$

② find $m\angle A$

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$m\angle A + 25^\circ + 140^\circ \approx 180^\circ$$

$$\boxed{m\angle A \approx 15^\circ}$$

③ find side a

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 140^\circ}{38} = \frac{\sin 15^\circ}{a}$$

$$\frac{a \cdot \sin 140^\circ}{\sin 140^\circ} = \frac{38 \cdot \sin 15^\circ}{\sin 140^\circ}$$

$$a = \frac{38 \cdot \sin 15^\circ}{\sin 140^\circ}$$

$$\boxed{a \approx 15.3 \text{ units}}$$

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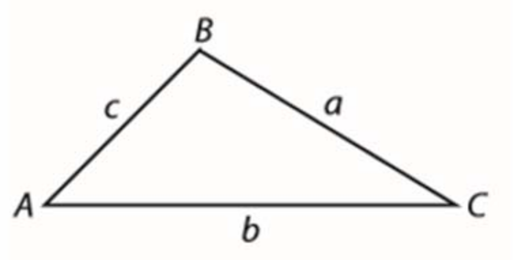
Closure

Suppose you are given $m\angle A$. To find c, what other measures do you need to know in order to be able to use the Law of Sines?



I need to know the measure of angle C and the measure of side a.