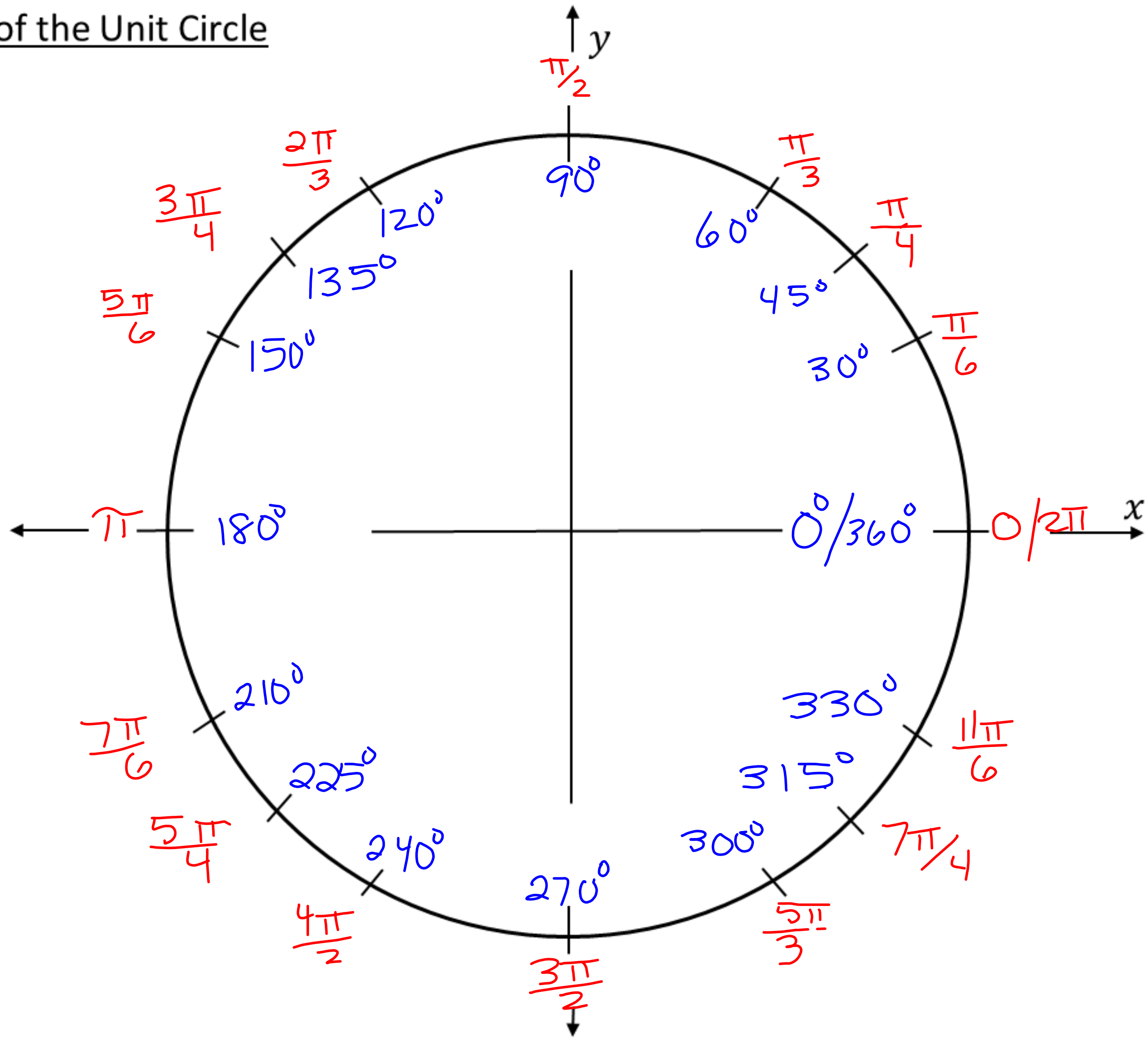
Objective: Complete and Use the Unit Circle

Concept

The Unit Circle is a circle with radius 1 unit. Trigonometry often uses the angles of the Unit Circle, especially in Calculus. The angles of the Unit Circle are the quadrantal angles and angles that are multiples of $30°/\frac{\pi}{6}$ radians and multiples of $45°/\frac{\pi}{4}$ radians.

Objective: Complete and Use the Unit Circle

Angles of the Unit Circle

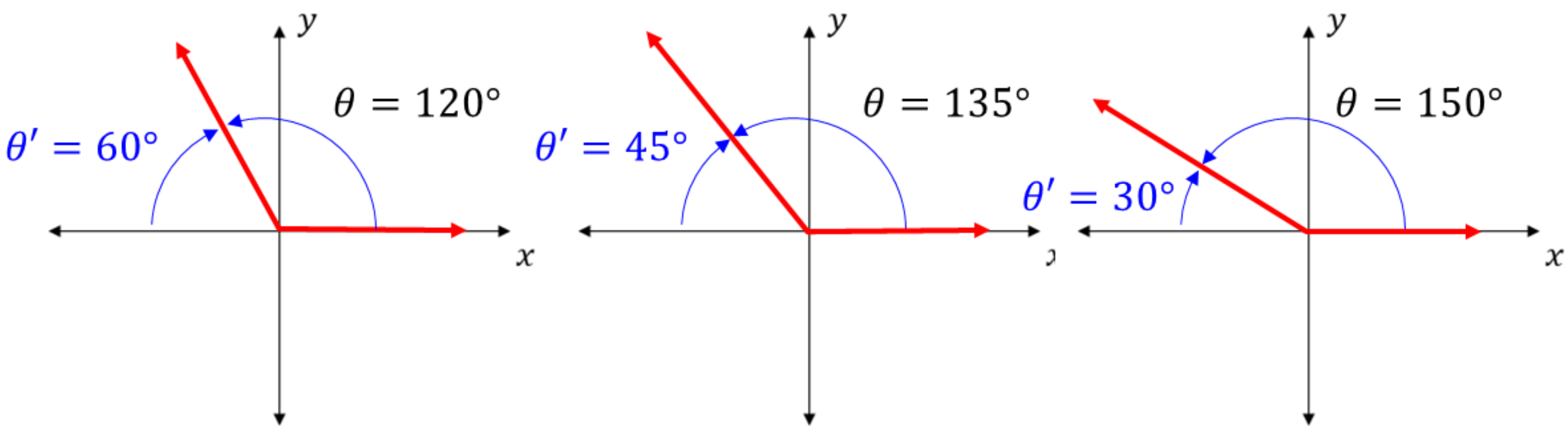


Objective: Complete and Use the Unit Circle

Concept

The reference angle, θ', of an angle θ in standard position is the acute angle formed by the x-axis and the terminal side of θ. The non-quadrantal angles of the Unit Circle have reference angles of $30°, 45°,$ or $60°$.

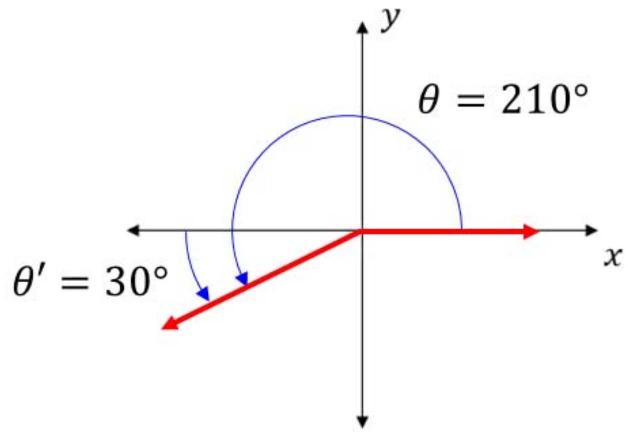
Reference Angles in the Second Quadrant

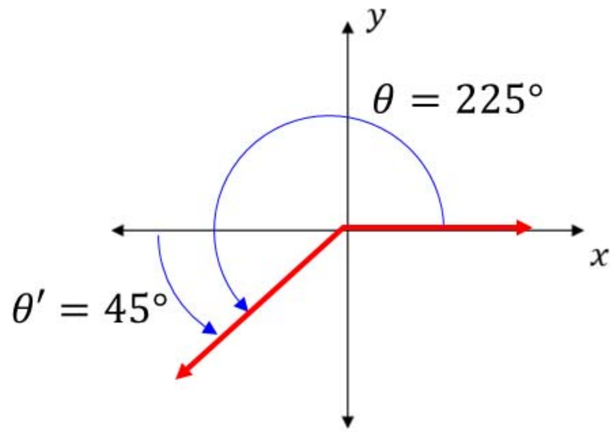


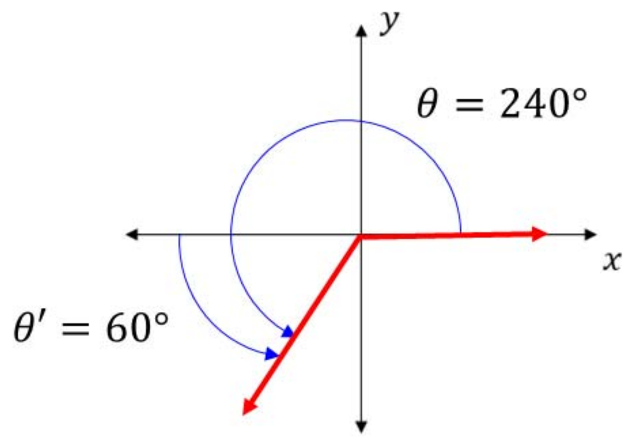
Objective: Complete and Use the Unit Circle

Concept

Reference Angles in the Third Quadrant



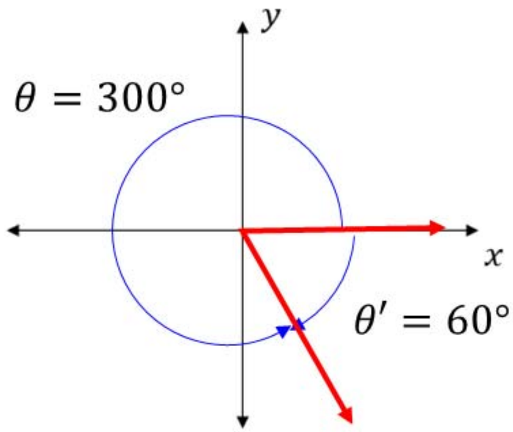


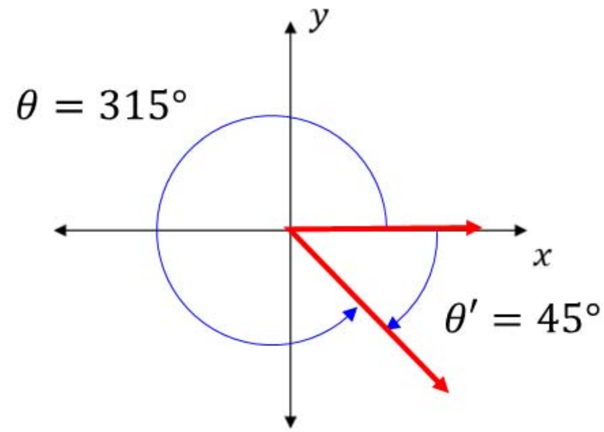


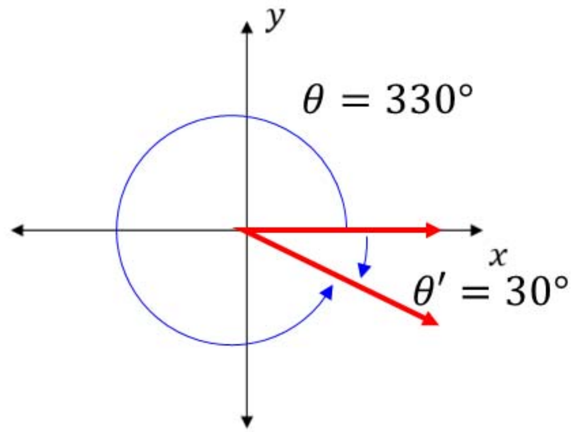
Objective: Complete and Use the Unit Circle

Concept

Reference Angles in the Fourth Quadrant



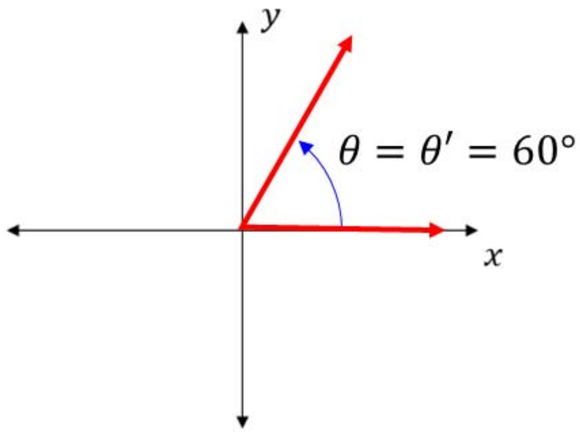


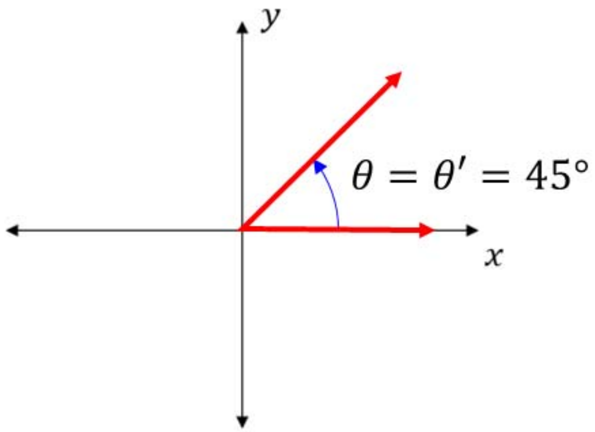


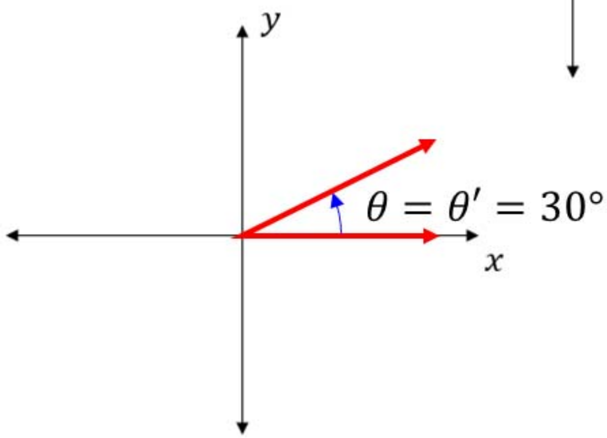
Objective: Complete and Use the Unit Circle

Concept

Reference Angles in the First Quadrant



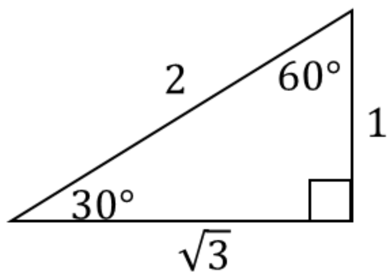


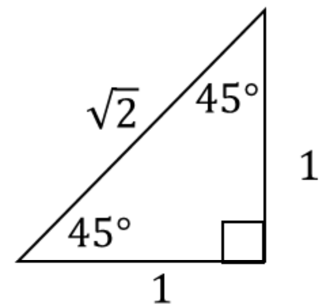


Objective: Complete and Use the Unit Circle

Concept

The $30° - 60° - 90°$ and $45° - 45° - 90°$ right triangles are referred to as <u>Special Right Triangles</u>. Since the acute angles of these triangles are the reference angles for the angles of the Unit Circle, **the sine and cosine values of these angles are a critical part of the Unit Circle**.





$\sin 30° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

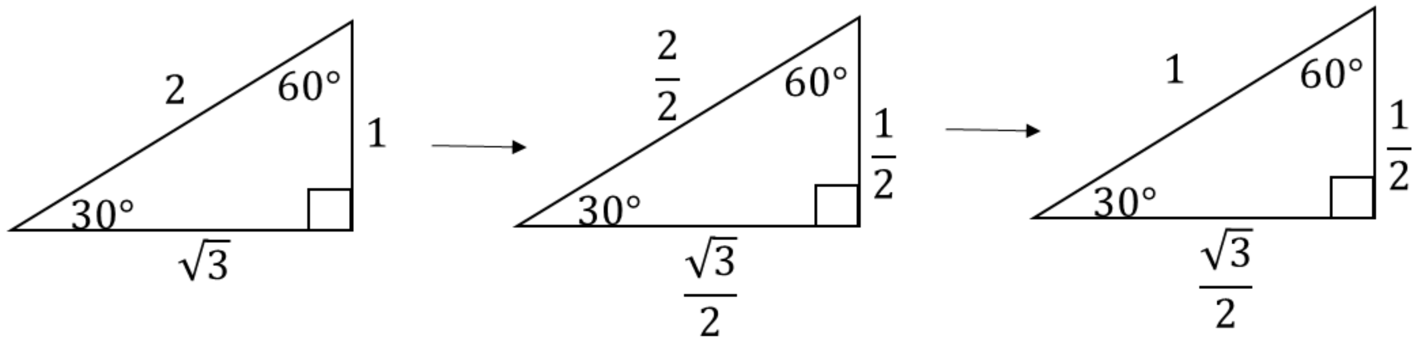
$\cos 60° = \frac{1}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Objective: Complete and Use the Unit Circle

Concept

You may recall that similar triangles have congruent angles and proportional sides. Dividing all side measures of the $\mathbf{30° - 60° - 90°}$ **triangle** by 2 results in a triangle **with a hypotenuse of 1 unit**. **The sine and cosine of the acute angles are unchanged.**



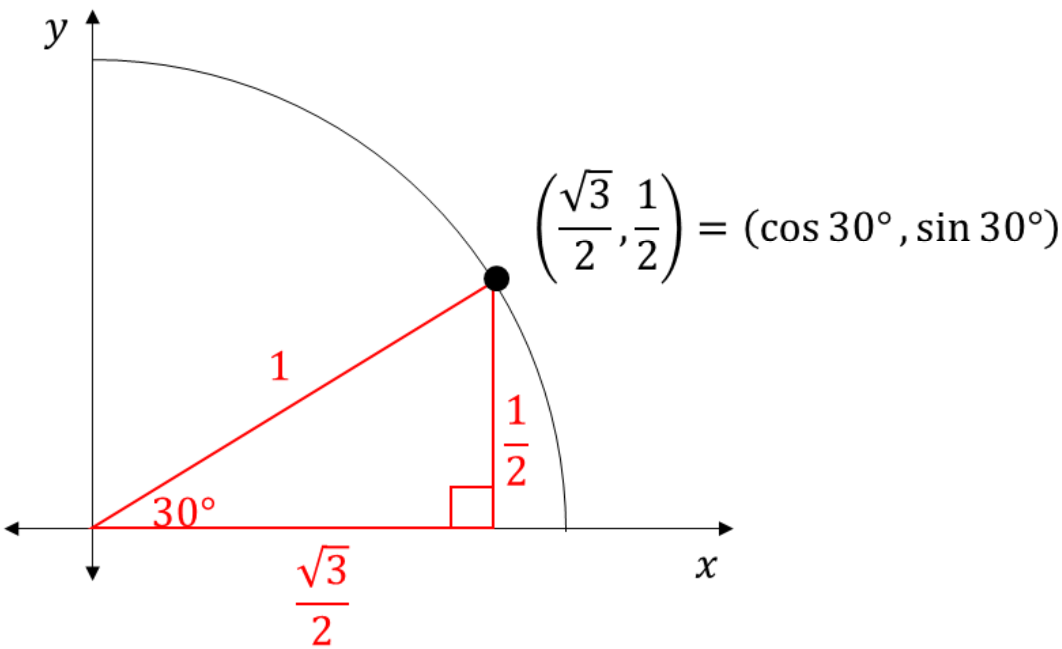
$\sin 30° = \frac{1}{2}$ $\qquad$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos 60° = \frac{1}{2}$

Objective: Complete and Use the Unit Circle

Concept

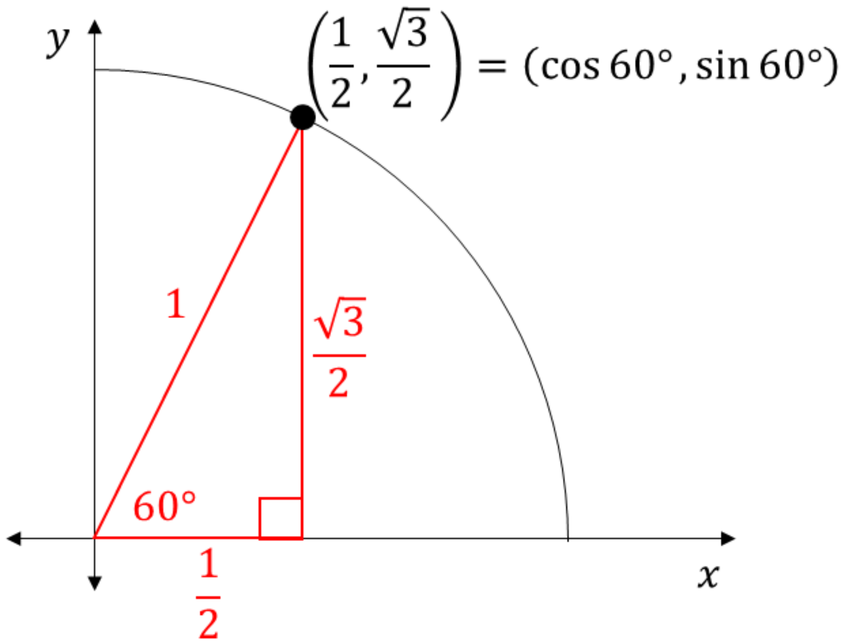
Placing the $\mathbf{30°} - \mathbf{60°} - \mathbf{90°}$ triangle with a central angle of $30°$ and a hypotenuse of 1 unit in the first quadrant of the Unit Circle shows us that the sine and cosine of the acute angle correspond to the point where the terminal side of the angle intersects the Unit Circle.



Objective: Complete and Use the Unit Circle

Concept

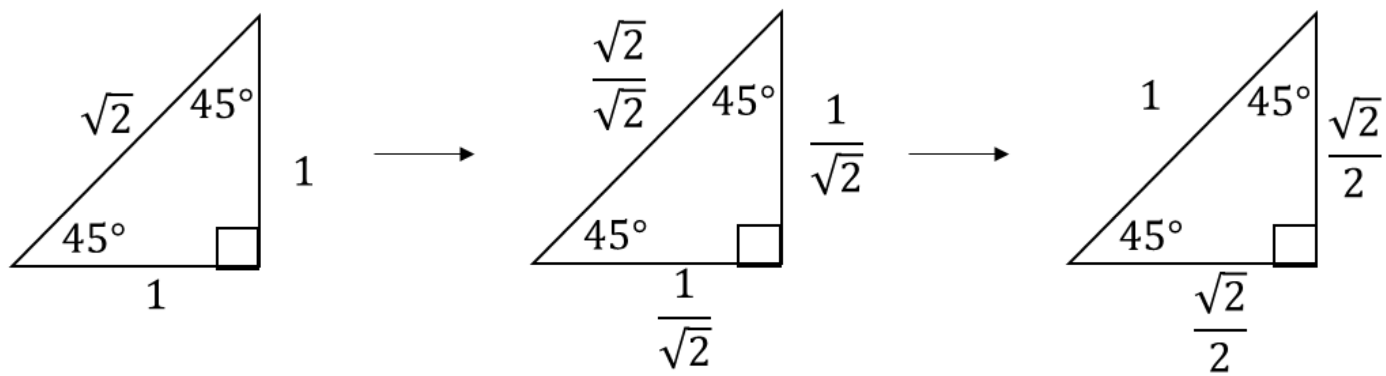
Placing the $\mathbf{30°-60°-90°}$ triangle with a central angle of $60°$ and a hypotenuse of 1 unit in the first quadrant of the Unit Circle shows us that the sine and cosine of the acute angle correspond to the point where the terminal side of the angle intersects the Unit Circle.



Objective: Complete and Use the Unit Circle

Concept

You may recall that similar triangles have congruent angles and proportional sides. Dividing all side measures of the $\mathbf{45° - 45° - 90°}$ **triangle** by $\sqrt{2}$ results in a triangle **with a hypotenuse of 1 unit**. **The sine and cosine of the acute angles are unchanged.**



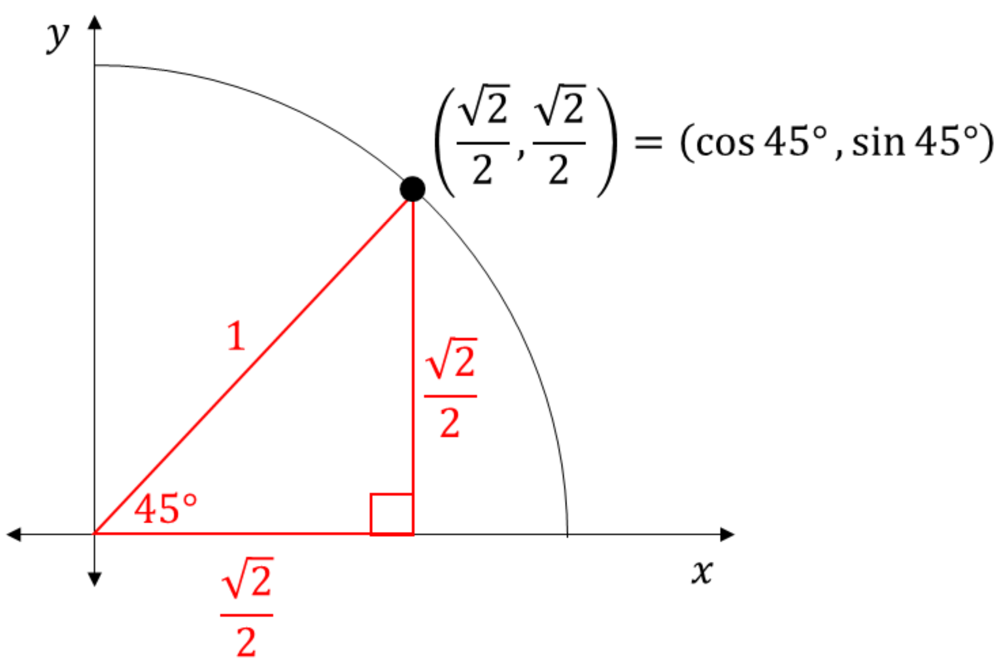
$$\sin 45° = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{\sqrt{2}}{2}$$

Objective: Complete and Use the Unit Circle

Concept

Placing the $\mathbf{45° - 45° - 90°}$ triangle with a central angle of $45°$ and a hypotenuse of 1 unit in the first quadrant of the Unit Circle shows us that the sine and cosine of the acute angle correspond to the point where the terminal side of the angle intersects the Unit Circle.



Objective: Complete and Use the Unit Circle

Concept

This means that for any angle, θ, in the Unit Circle the coordinates of the point where the terminal side intersects the circle correspond to $(\cos\theta, \sin\theta)$.

For each quadrant, identify whether $\sin\theta$ and $\cos\theta$ are positive or negative.

Q II

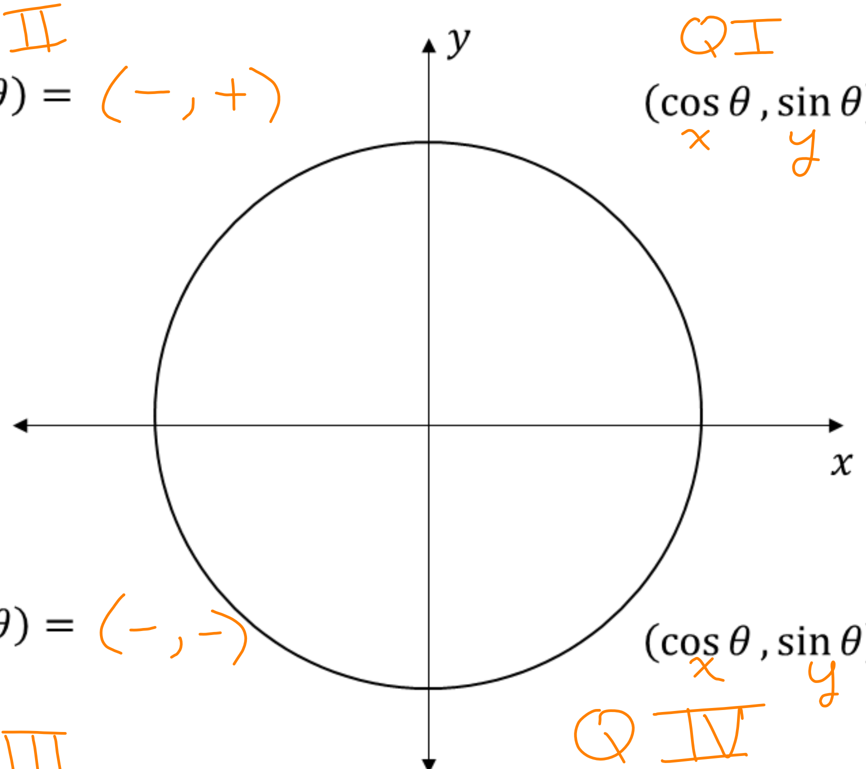
$(\cos\theta, \sin\theta) = (-, +)$

x y

Q I

$(\cos\theta, \sin\theta) = (+, +)$

x y



$(\cos\theta, \sin\theta) = (-, -)$

x y

Q III

$(\cos\theta, \sin\theta) = (+, -)$

x y

Q IV

Objective: Complete and Use the Unit Circle

Concept

Combining what we have learned about reference angles and the sine and cosine of the $30°, 45°,$ and $60°$ angles, complete the tables.

Unit Circle Angles with a Reference Angle of 30°

	Quadrant I $0° < \theta < 90°$	Quadrant II $90° < \theta < 180°$	Quadrant III $180° < \theta < 270°$	Quadrant IV $270° < \theta < 360°$
Angle of Rotation, θ	$30°$	$150°$	$210°$	$330°$
$\sin\theta$ y	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\cos\theta$ x	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$

Objective: Complete and Use the Unit Circle

Concept

Combining what we have learned about reference angles and the sine and cosine of the $30°, 45°,$ and $60°$ angles, complete the tables.

Unit Circle Angles with a Reference Angle of $45°$

	Quadrant I $0° < \theta < 90°$	Quadrant II $90° < \theta < 180°$	Quadrant III $180° < \theta < 270°$	Quadrant IV $270° < \theta < 360°$
Angle of Rotation, θ	$45°$	$135°$	$225°$	$315°$
$\sin\theta$ y	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\cos\theta$ x	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

Objective: Complete and Use the Unit Circle

Concept

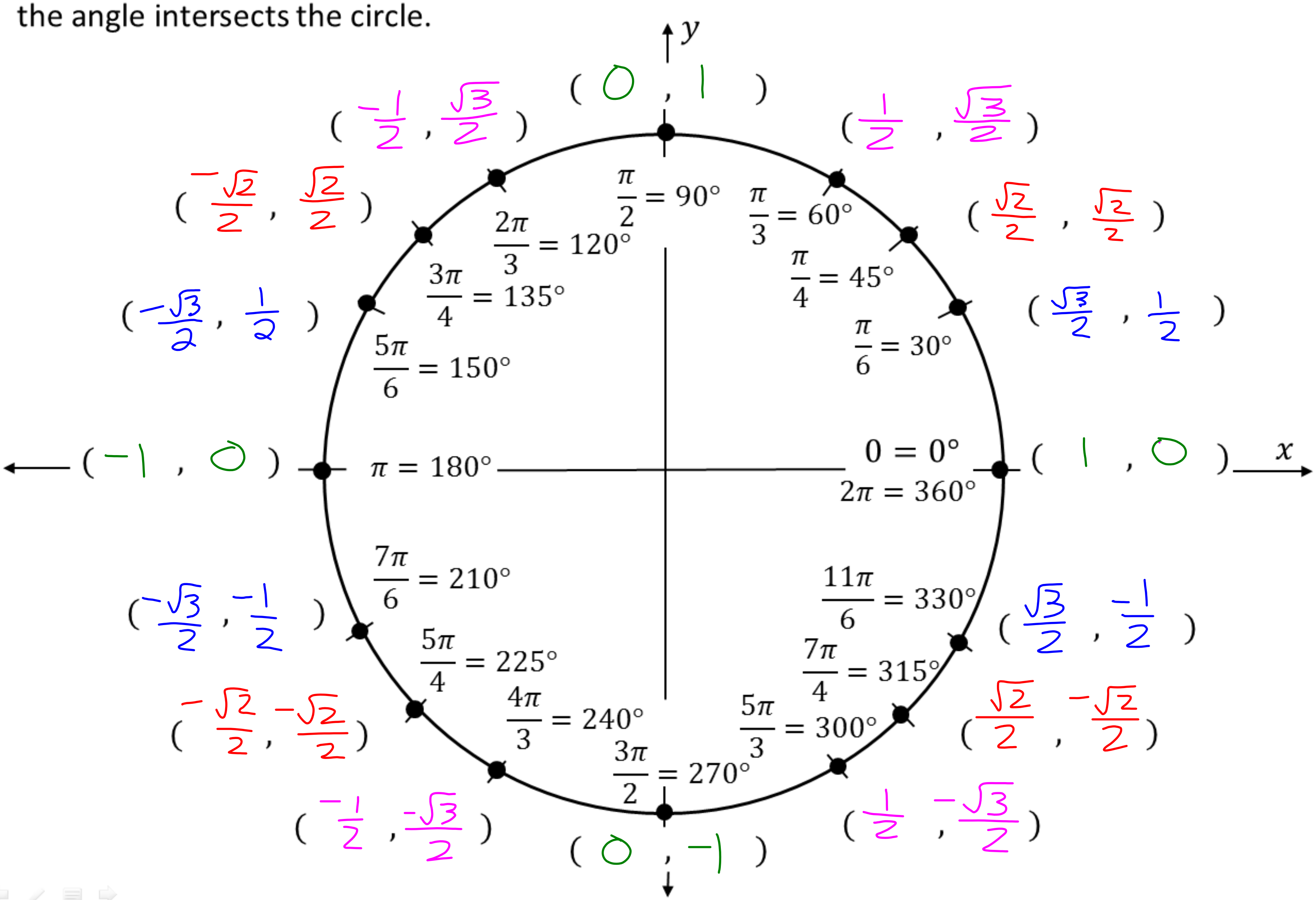
Combining what we have learned about reference angles and the sine and cosine of the $30°, 45°,$ and $60°$ angles, complete the tables.

Unit Circle Angles with a Reference Angle of $60°$

	Quadrant I $0° < \theta < 90°$	Quadrant II $90° < \theta < 180°$	Quadrant III $180° < \theta < 270°$	Quadrant IV $270° < \theta < 360°$
Angle of Rotation, θ	$60°$	$120°$	$240°$	$300°$
$\sin\theta$ y	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
$\cos\theta$ x	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

Objective: Complete and Use the Unit Circle

Complete the Unit Circle by Filling in the Coordinates of the points where the terminal side of the angle intersects the circle.



Objective: Complete and Use the Unit Circle

Closure

An angle θ intersects the Unit Circle at the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Explain what each coordinate of the point represents.

The x-coordinate of $\frac{1}{2}$ represents the cosine of θ, and the y-coordinate of $-\frac{\sqrt{3}}{2}$ represents the sine of θ.