Objective: Solve systems of equations.

Concept

Solving a System of Equations Using Substitution

1. Solve at least one function/equation for y, if necessary.
2. Substitute the expression equal to y into the other equation for the y variable.
3. Solve the new equation for x.
4. Find the corresponding y value.
5. Check each solution to verify its validity.
6. Write any valid solution as an ordered pair, (x, y).

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Ex) Solve the system using the substitution method.

$$\begin{cases} x + y = 2 \\ y = \sqrt{x} \end{cases}$$

① $y = \sqrt{x}$

$x + y = 2$

② $x + \sqrt{x} = 2$

$\sqrt{x} = 2 - x$

$(\sqrt{x})^2 = (2 - x)^2$

$x = 4 - 4x + x^2$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x - 4 = 0$ or $x - 1 = 0$

$\cancel{x = 4}$ $\qquad x = 1$ ✓

③ $y = ?$ when $x = 1$

$x + y = 2$

$1 + y = 2 \rightarrow y = 1$

$y = \sqrt{x} \rightarrow y = \sqrt{1} \rightarrow y = 1$

④ The solution to the system is $\boxed{(1, 1)}$

(x, y)

check

$4 + \sqrt{4} \stackrel{?}{=} 2$

$4 + 2$

$6 \neq 2$

$1 + \sqrt{1} = 2$

$1 + 1$

$2 = 2$

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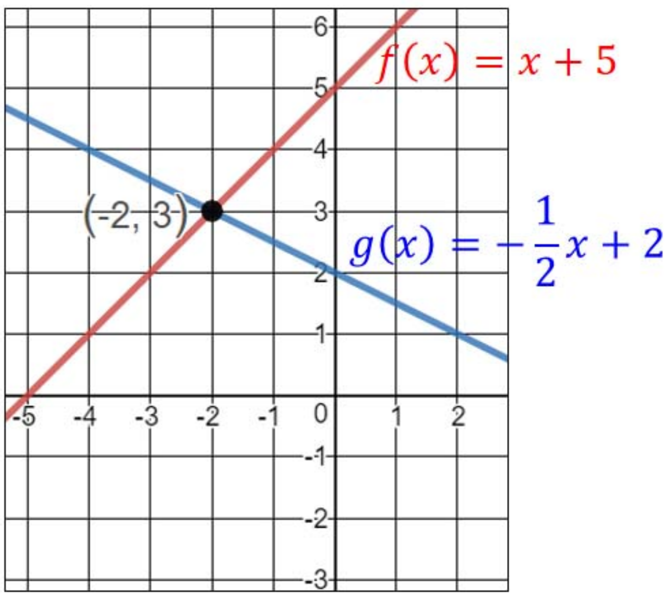
Concept

Two Functions are equal at any x value that produces the same y value in both functions. This would be the same value(s) as the x coordinate of any point of intersection between the graphs of the two functions.

To determine where two functions are equal, use the substitution method. Set the functions equal to one another and solve the resulting equation. Check the validity of the solution(s) in the original functions.



$$f(x) = g(x)$$
$$x + 5 = -\frac{1}{2}x + 2$$
$$\frac{3}{2}x = -3$$
$$x = -2$$

The functions $f(x)$ and $g(x)$ are equal at $x = -2$.

Objective: Solve systems of equations.

Ex) What value(s) of the independent variable (x) will produce the same function value (y) for the functions $f(x) = \sqrt{2x^2 + 10}$ and $g(x) = 2x$?

① system

$$\begin{cases} f(x) = \sqrt{2x^2+10} \\ g(x) = 2x \end{cases} \rightarrow \begin{cases} y = \sqrt{2x^2+10} \\ y = 2x \end{cases}$$

②

$$y = \sqrt{2x^2+10}$$

$$y = 2x$$

$$\sqrt{2x^2+10} = 2x$$

③

$$\left(\sqrt{2x^2+10}\right)^2 = (2x)^2$$

$$2x^2 + 10 = 4x^2$$

$$10 = 2x^2$$

$$5 = x^2$$

$$\pm\sqrt{5} = \sqrt{x^2}$$

$$x = \cancel{-\sqrt{5}}, \sqrt{5} \checkmark$$

check

$$\sqrt{2(-\sqrt{5})^2+10} \stackrel{?}{=} 2(-\sqrt{5})$$

$$2 \cdot 5$$

$$\sqrt{10+10}$$

$$\sqrt{20}$$

$$2\sqrt{5} \neq -2\sqrt{5}$$

$$\sqrt{2(\sqrt{5})^2+10} \stackrel{?}{=} 2(\sqrt{5})$$

$$\sqrt{20}$$

$$2\sqrt{5} = 2\sqrt{5}$$

④ conclusion

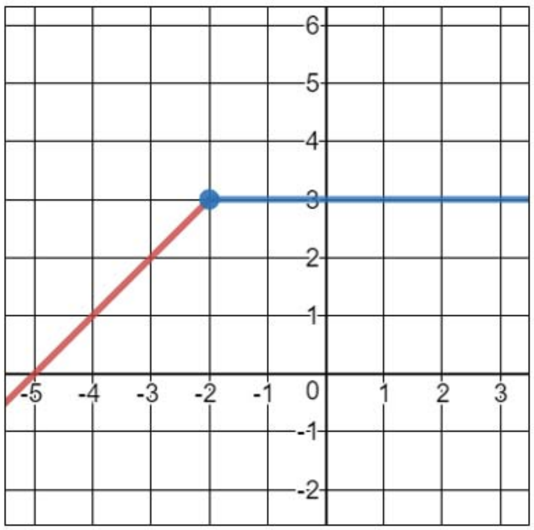
The functions have the same value when $x = \sqrt{5}$.

Objective: Solve systems of equations.

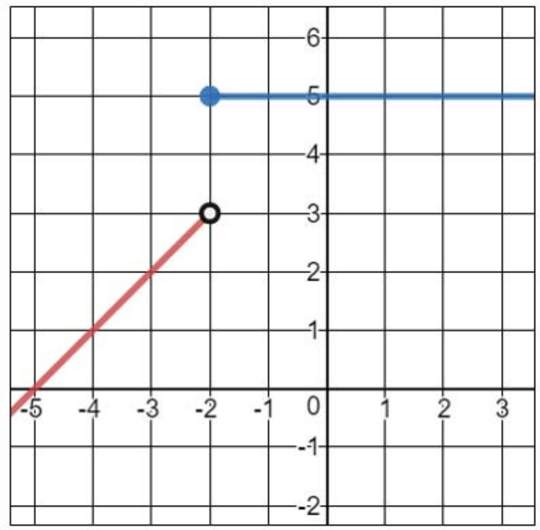
Concept

Piecewise functions may be continuous (unbroken) or discontinuous (broken).

Continuous Function



Discontinuous Function



Objective: Solve systems of equations.

Ex) For what value(s) of a will the piecewise function be continuous?

$$f(x) = \begin{cases} 4x - a; & x \geq 1 \\ \sqrt{ax - 2}; & x < 1 \end{cases}$$

For the function to be continuous, the y values must be the same where the domain separates. (Here that is for $x = 1$.)

Therefore,

1. Let $x = 1$ in both pieces and simplify.
2. Set the pieces equal to one another and solve for a.
3. Check the a values to determine if they are valid. State your conclusion.

1 let $x = 1$

$f(x) = 4x - a$
$4(1) - a$
$f(x) = 4 - a$
$y_1 = 4 - a$

$f(x) = \sqrt{ax - 2}$
$\sqrt{a(1) - 2}$
$f(x) = \sqrt{a - 2}$
$y_2 = \sqrt{a - 2}$

2 $y_1 = y_2$

$4 - a = \sqrt{a - 2}$

$(4 - a)^2 = (\sqrt{a - 2})^2$

$16 - 8a + a^2 = a - 2$

$a^2 - 9a + 18 = 0$

$(a - 6)(a - 3) = 0$

$a - 6 = 0, \quad a - 3 = 0$

$\cancel{a = 6} \qquad a = 3$ ✓

check.
$4 - a = \sqrt{a - 2}$

$a = 6$
$4 - 6 = \sqrt{6 - 2}$
$-2 = \sqrt{4}$
$-2 \neq 2$

$a = 3$
$4 - 3 = \sqrt{3 - 2}$
$1 = \sqrt{1}$
$1 = 1$

3 conclusion: The function is continuous when $a = 3$.