Objective: Create polynomial functions using operations and composition.

Concept

Recall: Operations on functions are similar to operations on polynomial expressions. Each function is defined for all x values in the domain of both f and g. The final result of the simplified expression represents all possible values for the range.

Sum of f and g: $(f+g)(x) = f(x) + g(x)$

Difference of f and g: $(f-g)(x) = f(x) - g(x)$

Product of f and g: $(f \cdot g)(x) = f(x) \cdot g(x)$

Objective: Create polynomial functions using operations and composition.

Ex) Given the functions $f(x) = 5x^3 + 2x - 3$ and $g(x) = -2x^3 + 4x^2$, create the following function. Write the result in standard form.

$$(f + g)(x)$$

$(f+g)(x) = f(x) + g(x)$

$(f+g)(x) = (5x^3 + 2x - 3) + (-2x^3 + 4x^2)$

$(f+g)(x) = 5x^3 + 2x - 3 + -2x^3 + 4x^2$

$(f+g)(x) = 3x^3 + 4x^2 + 2x - 3$

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Practice) Given the functions $f(x) = x^4 - 3x^3 + 5x^2 - 7$ and $g(x) = -x^3 + 4x^2 + x$, create the following function. Write the result in standard form.

$$(g - f)(x)$$

$$(g - f)(x) = g(x) - f(x)$$
$$(g - f)(x) = (-x^3 + 4x^2 + x) - (x^4 - 3x^3 + 5x^2 - 7)$$
$$(g - f)(x) = -x^3 + 4x^2 + x - x^4 + 3x^3 - 5x^2 + 7$$
$$\boxed{(g - f)(x) = -x^4 + 2x^3 - x^2 + x + 7}$$

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Ex) Given the functions $f(x) = x^3 + 4x^2 - 5x$ and $g(x) = x + 2$, create the following function. Write the result in standard form.

$$(f \cdot g)(x)$$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$(f \cdot g)(x) = (x^3 + 4x^2 - 5x)(x + 2)$

or

$= (x+2)(x^3 + 4x^2 - 5x)$

$= x^4 + 4x^3 - 5x^2$

$+ 2x^3 + 8x^2 - 10x$

$(f \cdot g)(x) = x^4 + 6x^3 + 3x^2 - 10x$

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Practice) Given the functions $f(x) = 5x^2 + 2x - 3$ and $g(x) = x^3 + 4x$, create the following function. Write the result in standard form.

$$(g \cdot f)(x)$$

$$(g \cdot f)(x) = g(x) \cdot f(x)$$
$$(g \cdot f)(x) = (x^3 + 4x) \cdot (5x^2 + 2x - 3)$$
$$(g \cdot f)(x) = (x^3 + 4x)(5x^2 + 2x - 3)$$
$$(g \cdot f)(x) = 5x^5 + 2x^4 - 3x^3 + 20x^3 + 8x^2 - 12x$$
$$\boxed{(g \cdot f)(x) = 5x^5 + 2x^4 + 17x^3 + 8x^2 - 12x}$$

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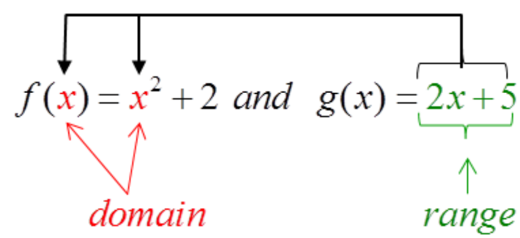
Concept

Composition is a mathematical process in which **one function, $f(x)$, uses the range (y values) of another function, $g(x)$, as its domain (x values).** This means the second function (which represents y values) is substituted into the first function for its x values.

The composition of $f(x)$ and $g(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$f(x) = x^2 + 2 \ \textit{and} \ g(x) = 2x + 5$$



Objective: Create polynomial functions using operations and composition.

Concept

Composition Notation

The composition of two functions, $f(x)$ and $g(x)$, can be written two ways.

1. $(f \circ g)(x)$
2. $f(g(x))$

Both notations can be read "$f\ of\ g\ of\ x$" or "$f\ composition\ g\ of\ x.$"

Procedure for the Composition of Functions

1. Substitute the second function into the first function, replacing all variables with the second function.
2. Simplify the expression.
3. Write the new function in standard form and using the composition notation.

Objective: Create polynomial functions using operations and composition.

Ex) Given $f(x) = x^4 + 3x^3 + x$ and $g(x) = 2x$, find the following.

$(f \circ g)(x)$

$= f(g(x))$

$= f(2x)$

$* \; f(x) = x^4 + 3x^3 + x$

$f(2x) = (2x)^4 + 3(2x)^3 + 2x$

$2^4 \cdot x^4 \qquad 3 \cdot 2^3 \cdot x^3 \qquad 3 \cdot 8$

$= 16x^4 + 24x^3 + 2x$

$(f \circ g)(x) = 16x^4 + 24x^3 + 2x$

Objective: Create polynomial functions using operations and composition.

Practice) Given $f(x) = x^4 + 3x^3 + x$ and $g(x) = 2x$, find the following.

$$(g \circ f)(x)$$

$$(g \circ f)(x) = g(f(x)) = g(x^4 + 3x^3 + x)$$

$$g(x) \quad = \quad 2x$$

$$\downarrow \qquad \downarrow$$

$$g(x^4 + 3x^3 + x) = 2(x^4 + 3x^3 + x)$$

$$\boxed{(g \circ f)(x) = 2x^4 + 6x^3 + 2x}$$

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Ex) Given $f(x) = x + 1$ and $g(x) = x^3 - 3x$, find the following.

$g(f(x))$

$= (g \circ f)(x)$

$= g(x+1)$

$* \; g(x) = x^3 - 3x$

$g(x+1) = (x+1)^3 - 3(x+1)$

$= (x+1)(x+1)(x+1) + -3(x+1)$

$(x+1)(x^2+2x+1)$

$x^3 + 2x^2 + x$

$+ x^2 + 2x + 1$

$= x^3 + 3x^2 + 3x + 1 - 3x - 3$

$g(f(x)) = x^3 + 3x^2 - 2$

Objective: Create polynomial functions using operations and composition.

Practice) Given $f(x) = x^4 + x^3 + 2x^2 - 2x$ and $g(x) = -3x$, find the following.

$$f(g(x))$$

$$f(g(x)) = (f \circ g)(x) = f(-3x)$$

$$f(x) = \quad x^4 \quad + \quad x^3 \quad + \quad 2x^2 \quad - \quad 2x$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$f(-3x) = (-3x)^4 + (-3x)^3 + 2(-3x)^2 - 2(-3x)$$

$$f(-3x) = 81x^4 + \; -27x^3 + \; 2(9x^2) - \; 2(-3x)$$

$$\boxed{f(g(x)) = 81x^4 - 27x^3 + 18x^2 + 6x}$$

Objective: Create polynomial functions using operations and composition.

Practice) Given $f(x) = x^2 - 1$ and $g(x) = 3x^2 - x + 2$, find the following.

$$(g \circ f)(x)$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 - 1)$$

$$(g \circ f)(x) = 3(x^2 - 1)^2 - (x^2 - 1) + 2$$

$$(g \circ f)(x) = 3(x^2 - 1)(x^2 - 1) - (x^2 - 1) + 2$$

$$(g \circ f)(x) = 3(x^4 - 2x^2 + 1) - x^2 + 1 + 2$$

$$(g \circ f)(x) = 3x^4 - 6x^2 + 3 - x^2 + 1 + 2$$

$$\boxed{(g \circ f)(x) = 3x^4 - 7x^2 + 6}$$

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Closure

A student was doing the following problem. Determine the student's error.

Given $f(x) = 3x^3 - x$ and $g(x) = x - 2$, find $(g \circ f)(x)$.

$$\begin{aligned}(g \circ f)(x) &= 3(x-2)^2 - (x-2) \\ &= 3(x^2 - 4x + 4) - x + 2 \\ &= 3x^2 - 12x + 12 - x + 2\end{aligned}$$

$$\boxed{(g \circ f)(x) = 3x^2 - 13x + 14}$$

The student's error is that they found $(f \circ g)(x)$, not $(g \circ f)(x)$. The student should have set up the problem the following way: $(g \circ f)(x) = (3x^3 - x) - 2$.