Objective: Graph polynomial functions from standard form

Concept

Polynomial Functions and End Behavior

The end behavior of a polynomial function is determined by two characteristics.

1. The **degree** (the highest power of the independent variable) of the function: **even or odd**
2. The **sign of the leading coefficient** (the constant factor of the first term when the function is written in standard form): **positive or negative**

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Concept

Odd Degree (x, x^3, x^5) Polynomial Functions

Positive Leading Coefficient

End Behavior

$as\ x \to -\infty, f(x) \to -\infty$

$as\ x \to \infty, f(x) \to \infty$

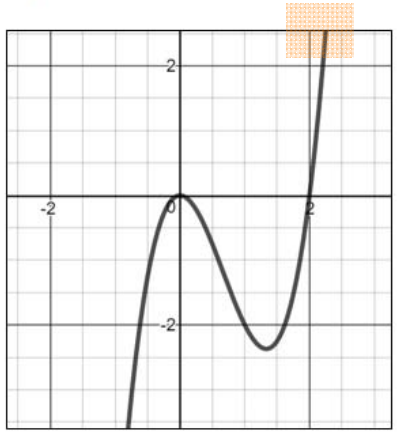
Negative Leading Coefficient

End Behavior

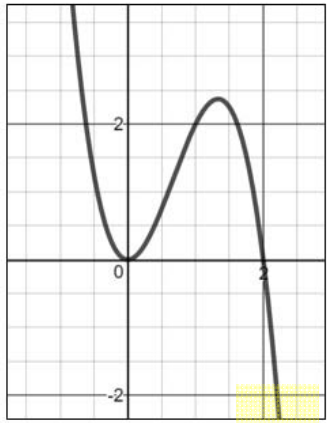
$as\ x \to -\infty, f(x) \to \infty$

$as\ x \to \infty, f(x) \to -\infty$

$f(x) = 2x^3 - 4x^2$



$f(x) = -2x^3 + 4x^2$



Objective: Graph polynomial functions from standard form

Concept

Even Degree (x^2, x^4, x^6) Polynomial Functions

Positive Leading Coefficient

End Behavior

$as\ x \to -\infty, f(x) \to \infty$

$as\ x \to \infty, f(x) \to \infty$

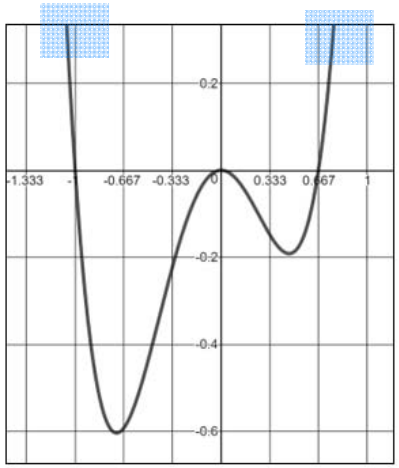
Negative Leading Coefficient

End Behavior

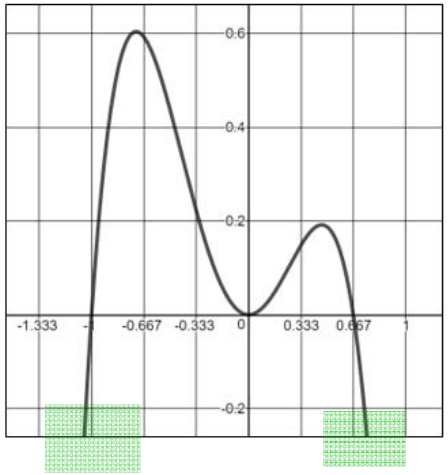
$as\ x \to -\infty, f(x) \to -\infty$

$as\ x \to \infty, f(x) \to -\infty$

$f(x) = 3x^4 + x^3 - 2x^2$



$f(x) = -3x^4 - x^3 + 2x^2$



Objective: Graph polynomial functions from standard form

Concept

Steps to Graph a Polynomial Function from Standard Form

1. Determine the end behavior using the first term.
2. Find the factored form of the function.
3. Find the zeros of the function. (Include multiplicity)
4. Sketch a smooth curve.

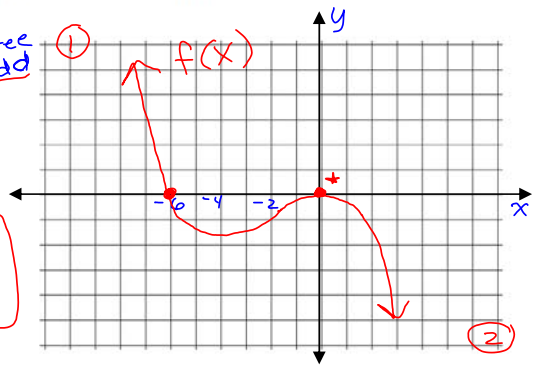
Objective: Graph polynomial functions from standard form

Ex) For each polynomial function: a) state the end behavior, b) state the exact values of the real zeros (include multiplicity), c) sketch the graph.

$f(x) = -2x^3 - 12x^2$

① first term = $-2x^3$ ③ degree is odd

* negative



② end behavior

as $x \to -\infty$, $f(x) \to +\infty$

as $x \to +\infty$, $f(x) \to -\infty$

② * factored form

$f(x) = -2x^3 - 12x^2$

* $f(x) = -2x^2(x+6)$

③ zeros

$0 = -2x^2(x+6)$

$\frac{-2x^2}{-2} = \frac{0}{-2}$

$x^2 = 0$

$\sqrt{x^2} = \pm\sqrt{0}$

$x = 0, 0$

$x + 6 = 0$

$-6 \quad -6$

$x = -6$

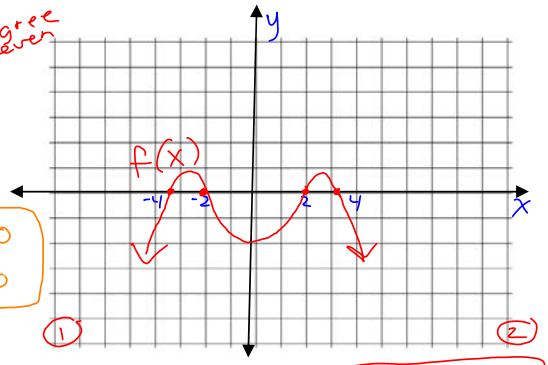
ⓑ zeros = $-6, 0, 0$

Objective: Graph polynomial functions from standard form

Ex) For each polynomial function: a) state the end behavior, b) state the exact values of the real zeros (include multiplicity), c) sketch the graph.

$f(x) = -x^4 + 15x^2 - 44$

① first term = $-1x^4$ (4) degree is even; negative



② end behavior

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to -\infty$

② factored form

$f(x) = -x^4 + 15x^2 - 44$

$f(x) = -1(x^4 - 15x^2 + 44)$

$f(x) = -1(x^2 - 11)(x^2 - 4)$

③ zeros

$0 = -1(x^2 - 11)(x^2 - 4)$

$-1 \neq 0$, $x^2 - 11 = 0$, $x^2 - 4 = 0$

$+11 \quad +11$ $\qquad$ $+4 \quad +4$

$x^2 = 11$ $\qquad$ $x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{11}$ $\qquad$ $\sqrt{x^2} = \pm\sqrt{4}$

$x = -\sqrt{11}, \sqrt{11}$ $\qquad$ $x = -2, 2$

$\approx -3.3, 3.3$

$\sqrt{9} \quad \sqrt{11} \quad \sqrt{16}$

$3 \quad \approx 3.3 \quad 4$

⑤ zeros

$= -\sqrt{11}, -2, 2, \sqrt{11}$

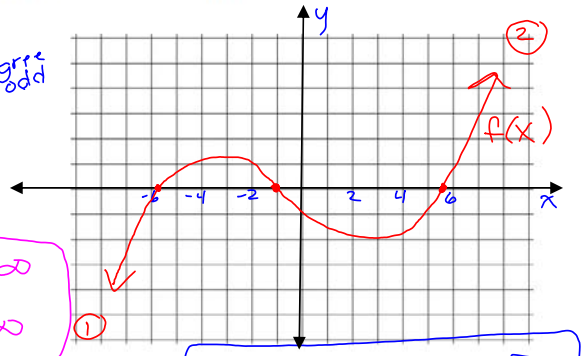
≈ -3.3 $\qquad$ ≈ 3.3

Objective: Graph polynomial functions from standard form

Ex) For each polynomial function: a) state the end behavior, b) state the values of the real zeros (include multiplicity), c) sketch the graph.

$f(x) = x^3 + x^2 - 34x - 34$

① first term = $1x^3$ ③ degree is odd, positive



④ end behavior

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$

② factored form

$f(x) = x^3 + x^2 - 34x - 34$

$f(x) = x^2(x+1) - 34(x+1)$

$f(x) = (x+1)(x^2 - 34)$

③ zeros

$0 = (x+1)(x^2 - 34)$

$x + 1 = 0$, $-1 \quad -1$

$x = -1$

$x^2 - 34 = 0$

$+34 \quad +34$

$x^2 = 34$

$\sqrt{x^2} = \pm\sqrt{34}$

$x = -\sqrt{34}, \sqrt{34}$

ⓑ zeros = $-\sqrt{34}, -1, \sqrt{34}$

≈ -5.8 ≈ 5.8

$\sqrt{25}$ $\sqrt{34}$ $\sqrt{36}$

5 ≈ 5.8 6