Objective: Graph quartic functions using transformations

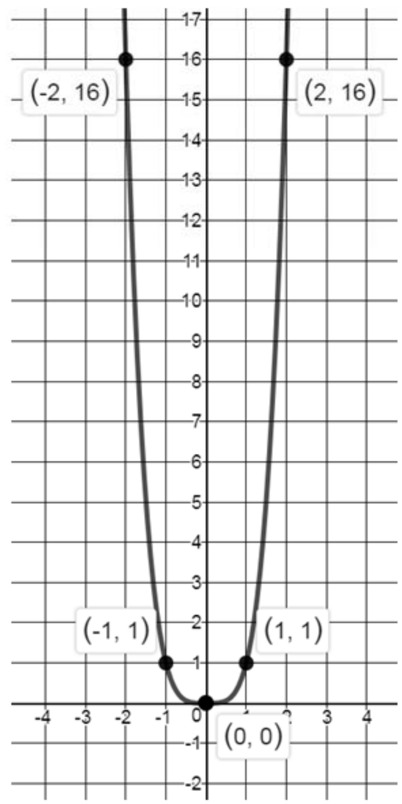
Concept

The parent function of the family of quartic functions is $f(x) = x^4$.
A quartic function in the form $f(x) = a(x-h)^4 + k$ can be graphed using transformations using the key points of the parent function.

For $f(x) = x^4$, the point $(0,0)$ is called the vertex. This point is only affected by translations. All other points are affected by all types of transformations.

x	$f(x) = x^4$
-2	$(-2)^4 = 16$
-1	$(-1)^4 = 1$
0	$0^4 = 0$
1	$1^4 = 1$
2	$2^4 = 16$

$f(x) = x^4$



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Concept

Recall: **The order in which transformations should be performed follow the Order of Operations.** Transformations that involve multiplication should be done first (reflections, stretches, compressions). Transformations that involve addition should be done second (translations right/left/up/down). There are exceptions and variations to this procedure, but this procedure always works.

One Procedure for Graphing a Quartic Function Using Transformations

1. Determine the transformations.
2. Translate $(0,0)$ to determine the new vertex.
3. Perform any reflection, stretch, and/or compression on the other key points of the parent function and then translate these points.
4. Draw a smooth curve through the points.

Objective: Graph quartic functions using transformations

Ex) A) Graph using transformations. B) Determine the zeros or the intervals of consecutive integers in which the zeros occur.

$g(x) = 2(x - 1)^4 + 2$ → parent $f(x) = x^4$

a

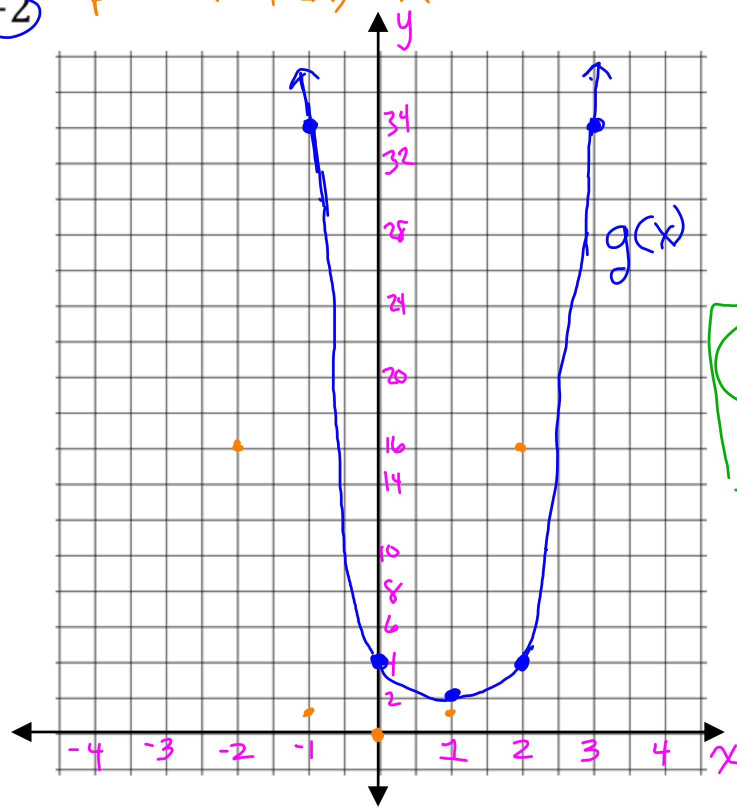
$a = 2$ no refl.

$|a| = |2| = 2 \geq 1$

vert. stretch

$h = 1$ right 1

$k = 2$ up 2



B) There are no real zeros.

Objective: Graph quartic functions using transformations

Ex) A) Graph using transformations. B) Determine the zeros or the intervals of consecutive integers in which the zeros occur.

$$d(x) = -\frac{1}{2}x^4 + 4$$

a   k

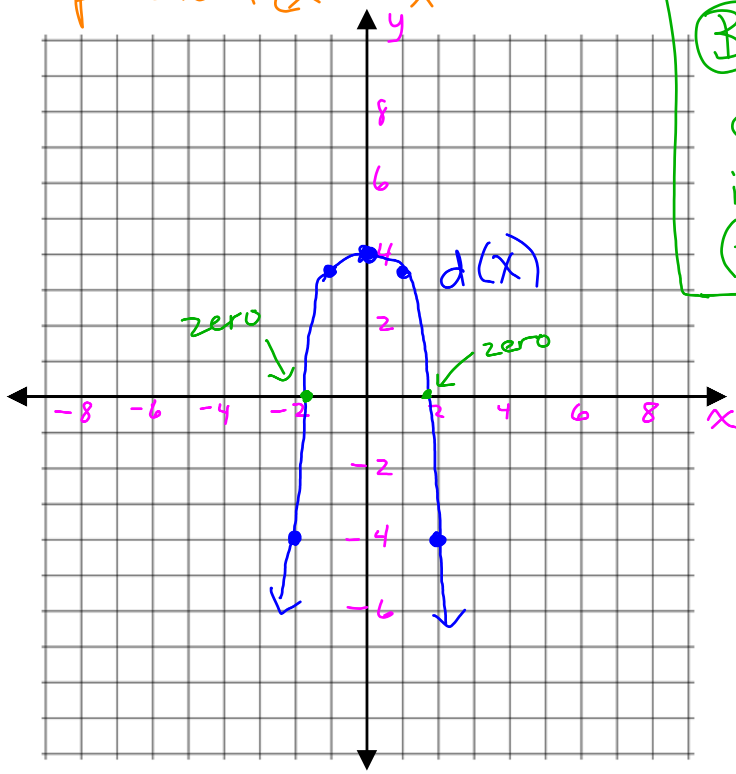
parent $f(x) = x^4$

$a = \frac{-1}{2}$ x-axis refl.

$|a| = |\frac{-1}{2}| = \frac{1}{2} < 1$

vert. comp.

$k = 4$ up 4



B) The zeros of d(x) are in the intervals (−2,−1) and (1,2).

Objective: Graph quartic functions using transformations

Concept

Given a parent function $f(x)$, $g(x) = af(x-h)+k$ and $g(x) = f\left(\frac{1}{b}(x-h)\right)+k$ can be graphed by identifying the transformations and then transforming the key points of $f(x)$.

Objective: Graph quartic functions using transformations

Ex) Given the parent function $f(x) = x^4$, graph $g(x)$ using transformations.

$g(x) = f(2x) - 1$ k

$\frac{1}{b}$

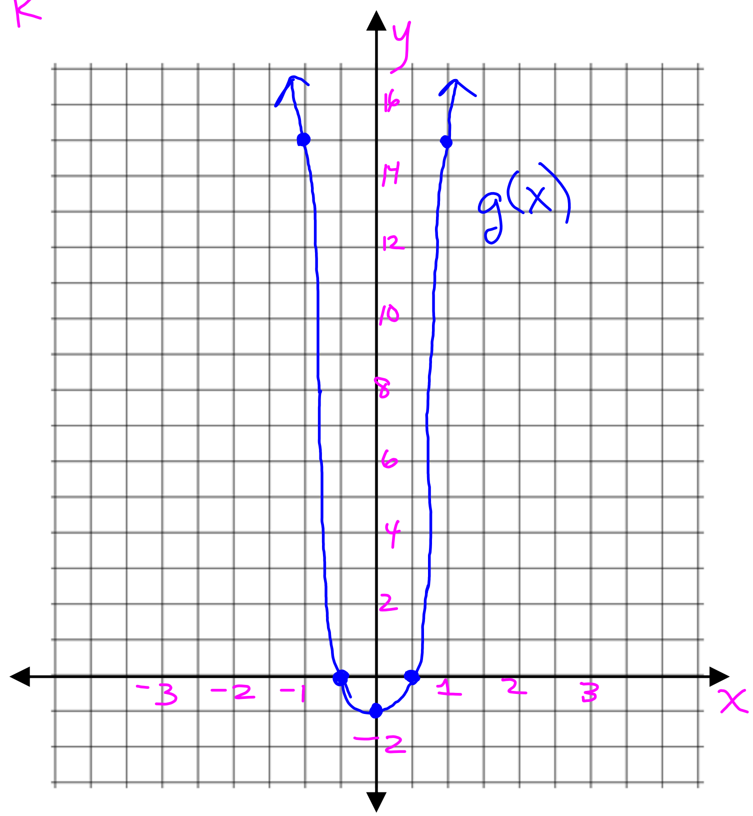
$\frac{1}{b} = 2$

$b = \frac{1}{2}$ no refl.

$|b| = |\frac{1}{2}| = \frac{1}{2} \leq 1$

horiz. comp.

$k = -1$ down 1



Objective: Graph quartic functions using transformations

Ex) Given the parent function $f(x) = x^4$, graph $g(x)$ using transformations.

$$g(x) = f\left(-\frac{1}{2}(x+5)\right) - 6$$

$\frac{1}{b}$

$\frac{1}{b} = \frac{-1}{2}$

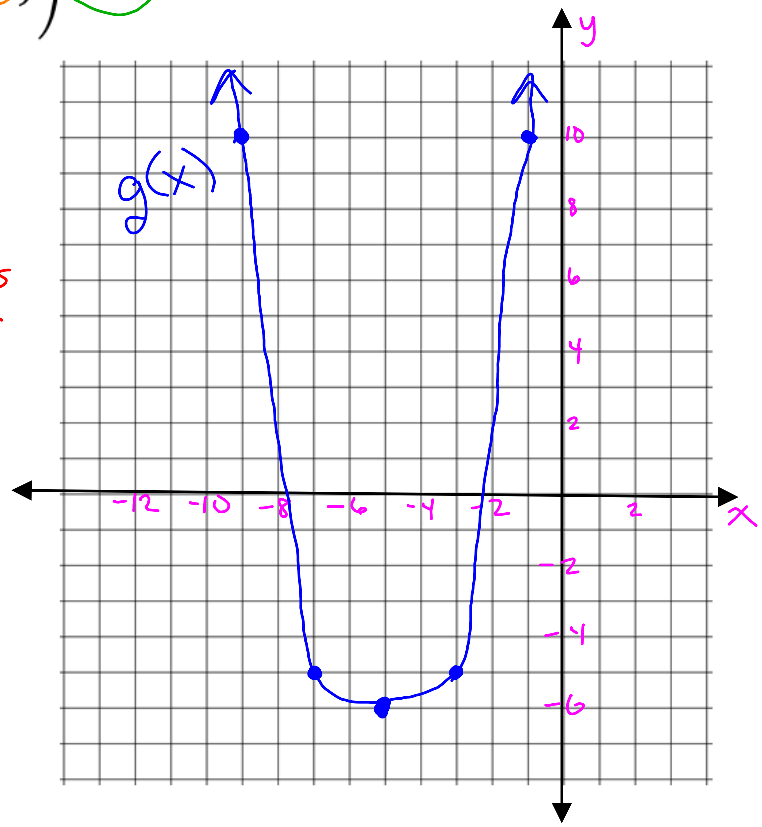
$b = -2$ y-axis refl.

$|b| = |-2| = 2 > 1$

horiz. stretch

$h = -5$ left 5

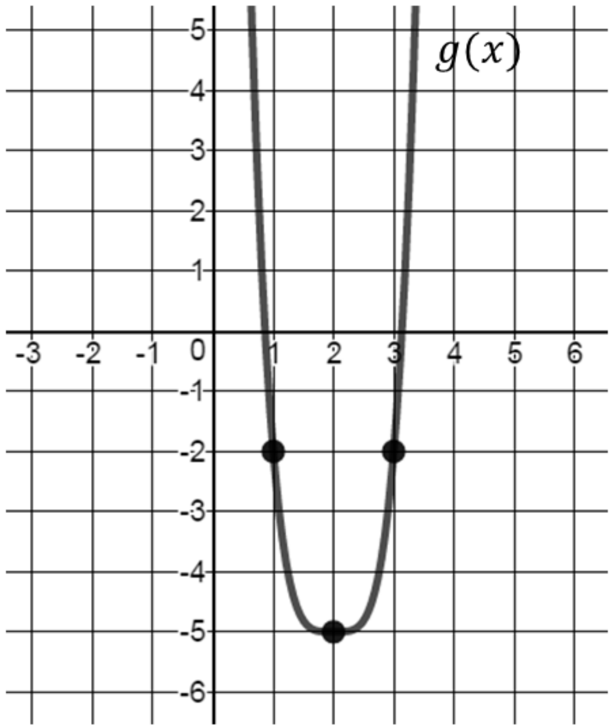
$k = -6$ down 6



Objective: Graph quartic functions using transformations

Closure

Which function could represent the graph of $g(x)$?



1. $g(x) = (x-2)^4 - 5$
2. $g(x) = (x+2)^4 - 5$
3. $g(x) = 2(x-2)^4 - 5$
4. $g(x) = 2(x+2)^4 - 5$
5. $\boxed{g(x) = 3(x-2)^4 - 5}$
6. $g(x) = 3(x+2)^4 - 5$